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A FRAMEWORK FOR MATHEMATICS INQUIRY-BASED CLASSROOM PRACTICE: THE CASE OF CÉLIA. [1]

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This paper has been developed in the context of the research project P3M Professional Practices of Mathematics Teachers. One of its main aims is to propose a framework for mathematics inquiry-based classroom practice, combining theoretical perspectives and the analysis of the teaching practice of experienced teachers that regularly conduct inquiry-based teaching of mathematics – in this paper, we focus on Célia case, a primary teacher teaching a 4th grade class. The framework adopts a four phases model for the lesson structure: 1) Introduction of the task; 2) Development of the task; 3) Discussion of the task; and 4) Systematization of the mathematical learning. For each phase, we describe the actions that the teacher intentionally performs with two main interrelated purposes: to promote the mathematical learning of the students and to manage the students and the class as a whole.
Key words: inquiry-based classroom practice, intentions and actions of the teacher, four phases model of the lesson.

CONTEXT AND AIM OF THE STUDY

In Portugal, as in many countries, the current mathematics curriculum orientations pose challenging goals to students' learning. However, they also pose significant challenges to teachers' practices in classroom. The traditional way of teaching mathematics, centred on teacher exposition of the topics followed by the repetition of procedures by the students, has been dominant (Franke, Kazemi, & Battey, 2007) but it is no longer adequate to deal with all the exigencies. Nowadays, students need to have opportunities to evolve themselves in significant mathematical tasks that allow them to reason mathematically about important mathematical ideas and to make sense of the knowledge that arises from the collective discussion of the tasks (NCTM, 2000; Ponte, 2005). This requires from the teacher an inquiry-based approach to teaching, centred on the work done by the students when they

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engage in the mathematical exploration of valuable tasks (Ponte, 2005; Stein, Engle, Smith, & Hughes, 2008). Our aim is to understand the mathematics inquiry-based classroom practice, identifying what are the main intentions of the teacher in each phase of the lesson and describing with detail the actions she/he performs while teaching. We think that the elaboration of a framework for mathematics inquiry-based classroom practice constitutes a useful resource to use in teacher education programs and to promote teacher development.

THEORETICAL PERSPECTIVES

The inquiry-based approach to mathematics teaching requires from the teacher much more than the identification and selection of the tasks. The selection of a valuable task is very important because it conveys an opportunity for learning but once the task is selected, it is crucial to think about how to explore its potentialities to promote mathematics learning, and to prepare to a complex activity in the classroom (Stein *et al.*, 2008).

A typical lesson in inquiry-based approach is generally structured in three or four phases: the "lauch phase", the "explore phase", and the "discuss and summarize phase" (Stein *et al.*, 2008). On the first phase, the teacher presents a mathematical task to the class. The task is often a problem or an investigation requiring interpretation. The teacher must ensure, in few minutes, that students understand what they are required to do and that they feel challenged to work on the task. The teacher also has to organize the class for work, establishing time for the different phases and setting up the resources to be used.

On the second phase, the teacher supports students autonomous work on the task either they are working individually or in small groups. The teacher needs to ensure that students engage in productive work. It is important that the comments and responses of the teacher to students' questions do not lower the level of cognitive demand of the task (Stein & Smith, 1998) and do not uniform the strategies and solutions in order to not frustrate the hypothesis of an interesting and challenging mathematical discussion for every student. The teacher needs to ensure that students prepare themselves to present their work to the whole-class and produce the adequate materials in time for the discussion phase. In the meanwhile, the teacher has to select from students productions the solutions she/he evaluates as positive contributions to the collective discussion and to sequence the order of its presentation by the students (Stein *et al*., 2008).

After that, the class returns to plenary for the discussion of the selected solutions of the students. The teacher has to orchestrate this discussion, not just managing students' interventions and interactions but also promoting the mathematical quality of students' explanations and discussion. She/he also needs to create and maintain a positive climate of genuine interest in the discussion, trying to guarantee the participation of all the students. It is very important that the discussion aims more than the comparison and confrontation of students' solutions. The teacher has a crucial role in helping students to summarize the main mathematical ideas that emerged from the discussion and institutionalize it, in ways that advance the mathematical learning of the whole class. This is a moment when teacher can introduce or systematize new concepts, synthesise procedures, reinforce key aspects of

processes like problem solving, representation, mathematical reasoning, mathematical communication (Ruthven, Hofmann, & Mercer, 2011; Stein *et al*., 2008).

Developing an inquiry-based approach is a complex practice for most of the teachers, namely in what concerns the orchestration of discussions (Franke, Kazemi & Battey, 2007; Stein *et al*., 2008). Teachers' practices can be viewed as the activities that they regularly conduct, taking into consideration their working context, and their meanings and intentions (Ponte & Chapman, 2006). We stress Lampert's vision that the "teaching practice is what teachers do, but it is more than how teachers behave with students or the actions of individual teachers; action is behaviour with meaning, and practice is action informed by a particular organizational context" (2004, p. 2). So, to describe and understand the teaching practice of a teacher it is important to explicit not only his/her actions but also the intentions that are embodied in her/his actions, the reasons she/he has to perform in certain ways.

It is also important to stress that teaching is a relational and multidimensional activity (Franke, Kazemi, & Battey, 2007). The relational dimension came from the relations that are established between the teacher, the students and the content, that can only be understood in relation one to another: "The teacher works to orchestrate the content, representations of the content, and the people in the classroom in relation to one another" (Franke, Kazemi, & Battey, 2007, p. 227). The multidimensionality dimension of teaching emerges from the diversity of scenarios and demands that are simultaneously present in classroom. The teacher needs to create an environment for the learning of her/his students, to orchestrate their participation so they relate to representations of the subject matter an to one another in particular ways, to elicit and interpret what students do and know and to provide them learning trajectories that allow them to develop mathematically. This requires from the teacher a continuously process of making decisions that combines teacher' knowledge, beliefs and goals (Franke, Kazemi, & Battey, 2007).

METHODOLOGICAL APPROACH

This paper reports to the first phase of P3M Project, dedicated to the construction of multimedia cases of teachers participants in the project. To construct the cases focused on teachers' practices, we adopt an interpretative approach for the investigation, considering the importance of capture teachers' perspectives to understand the actions they performed while teaching (Sowder, 2007).

We choose to work with experienced teachers that feel comfortable with the inquiry-based approach to mathematics teaching so they could provide a good context for the data collecting required for the development of the intended framework. We also choose to work with teachers of different levels, from primary teachers to secondary teachers, and focusing on classes about different mathematical topics so we can have a broader picture of the inquiry-based practice, independently of the contents being taught.

For each one of the teachers, data were collected in three moments, before, during and after the lessons. Before the lesson, we made an initial interview for understanding teacher planning and the main options for the development of the lesson, regarding the task and the

methodological orientations; we also registered the teacher anticipation of strategies and difficulties of her students. This interview was accompanied by the analysis of the plan of the lesson previously prepared by the teacher.

The data collection in the classroom involved the use of two video cameras to register the moments of work with whole class, as well as some interaction episodes between the teacher and the students when they worked autonomously on the development of the task. From that video collection, we selected the segments subjected to analysis and that became teaching episodes to use in the post lesson interview.

After the lesson, we made an interview of reflection on practice based on the analysis of episodes of the videotaped lesson, with the purpose of registering the teacher's explanations about the development of the lesson and the justifications of the her actions, confronting the planned and implemented lesson. In this post-interview, the watching of some selected teaching episodes helped the teacher to focus on particular events and not only to refer to superficial ideas or general guideline principles.

The planning and conduction of the interviews were based on the general a-priori categories that we considered, inspired by theoretical perspectives about the inquiry-based mathematics teaching practice, namely concerning the structure of the classroom and the two global purposes of promoting students learning and managing the class. This means that during the data collection (observation and interviews) we tried to obtain data that allow us to describe the teacher's actions and intentions for each one of the phases of the lesson, adapting and completing our categories to the reality of the teacher practice. For instance, at the beginning, we were not quite sure about adopting a three or four phases model for the lesson, but the analysis of the first case gave us conviction to distinguish two different phases after the development of the task by the students.

Following this method, we got a picture of the intentional actions of each teacher inquiry-based classroom practice. We hope that on a second phase of our work, a transversal analysis of the cases will allow us to elaborate a global framework for mathematics inquiry-based classroom practice, with the identification of a set of concrete actions that are important to consider in each phase of the lesson and also the reasons that justify these actions – and that provide a rationale for the framework.

THE INQUIRY-BASED TEACHING PRACTICE OF CÉLIA

In this communication we focus on Célia case, an experienced primary teacher teaching a 4th grade class, and we refer to the lesson where she explored the task "Cubes with Stickers" (appendix A), with the global objective of developing the algebraic reasoning of her students. The teacher selected this task in the sequence of another one that also provided the opportunity to generalize a rule by an expression with letters.

The lesson began with the presentation of the task by Célia to the whole class, followed by a period of students working in pairs, then a collective discussion of some different solutions proposed by students and finally a global discussion of the strategies used by the students to solve the task and of the different mathematical representations of the solutions they got. The

four phases were planed by Célia: her lesson plan was organized by "introduction of the task", "work in pairs", "collective discussion" and "systematization". Her plan also revealed, for each phase, a set of detailed actions for her to perform, most of them devoted to promote the mathematical learning of the students (by example, to put some specific questions to the students, to compare different specific solutions,…) and some others concerning the management of the class (by example, to organize students in pairs, to select the students solutions to be discussed, to provide transparencies and pens to prepare the presentations,…). She also registered the amount of time that she intended to devote to each phase of the lesson.

In the following we selected some excerpts of Célia post reflection interview, in which the teacher explains her options and her main intentions that allow us to interpret their actions in each phase on the lesson.

Introduction of the task

Célia explains very clearly her intentions when presenting the task to the class. She thinks that this moment is decisive for the god development of the mathematical work of the students. This Célia' concern is justified by the nature of the tasks that she chooses when she develops an inquiry approach in the classroom: challenging tasks, often problems or investigations that require interpretation. She tries to assure that the students understand well the context and the objectives of the task, listening attentively to their comments and questions:

> I guess it should be a presentation of a challenge and it should be an interpretation about what the challenge requires… I think that interpreting and understanding what it's needed will be the two objectives. (…) I have the preoccupation of dedicate enough time to them to understand and to put their questions and doubts, too… Questioning them and waiting for their doubts is also a way for me to realize that they understand.

In Célia' s view, if students do not understand the task, "they will create confusions and then… sometimes it is too late in the lesson to clarify and to recover them…". Besides to guarantee the comprehension of the task by the students, Célia also wants them to engage with the task and to accept/assume the challenge of solving it:

> It's to make them want the task, to predispose them to the task. It is not only to understand, but also to assume the task by their own… This is where the challenge also emerges… to assume the task as a matter of me that I want to solve.

Development of the task

Célia focus her reflection about this phase of the lesson on her supporting students autonomous work on the task. She confesses her difficulties in monitoring the progress of students' work, which requires, in her opinion, "a balance" between let them by their own and give them some orientation. But, above of all, she wants to gave them opportunity to solve the task by their own strategies:

> The aim is that they work on the task, that they work in different ways, the ways they are able (…) and I think that my role is an extremely difficult role.

The teacher tries to follow the reasoning of the students, asking them questions and listening to their explanations. Sometimes this is hard to accomplish because students reasoning's might not to be obvious:

> When I try to go from one pair to another, or from group to group, I try to understand how they are, what they see, how they are working – and I try to do this with questions and sometimes is not easy. (...) I try to follow their reasoning but it is not easy because sometimes… in some situation we are so far away… we see that the student is right... But how did he think?

Another concern of Célia is to not validate the correctness of student's strategies or responses, neither when they are right or wrong – although she knows that sometimes her facial expression reveals more than she wants. When she identifies groups that are working in a non productive way, she puts them questions with the intention of make them to reflect on their strategy, on their mistakes, and to adopt another more productive way of solving the task:

> If I evaluate that they are going for a completely wrong way, I try not to say: "This is not correct"... I try to say: "how did you start? Go back to the beginning..." For example, if they are mistaken for ten cubes, I ask them: "This way of thinking is the same you use for the three cubes?"... I try to make them reach a productive way....

But Célia also refers to the difficult parallel activity that she has to develop while monitoring student's work, concerning the organization of the discussion of the task. The teacher carefully chooses the student's strategies that she identifies as relevant contributes to the mathematical collective discussion that she wants to promote aiming to accomplish the specific mathematical purpose of the lesson:

> To choose which of those solutions are important for the collective discussion - and this is very difficult because... it requires almost a detachment that is difficult to manage in the classroom (...) with all those requests. (...) Of these eight different resolutions, which one interests me to discuss in this particular lesson?

She also tries to sequence the order of discussion of the selected solutions in function of her perceived potential for promote clarification and understanding of the task and to exemplify productive forms of representation, also concerning the mathematical purpose of the lesson.

> I have picked this one first because of the visualization that it allows to the students... This (other) was because of the image and the way they represented, they were very clear here, 1+1+1... so were the eight cubes to represent the different faces (...) This [other]… I thought that if there were any questions or if there were still problems, this was more an attempt to solve them because of its clarity.

Discussion of the task

Célia explicitly values this phase of the lesson because it constitutes an important opportunity of mathematical learning for all students in the classroom:

> It is extremely important because the lesson is not only the working in pairs, but it is... in terms of math class, what is the collective apprehension, what is to be discussed?

She asks the students authors of the selected solutions to expose their strategies and to explain their reasoning to the colleagues, and also to respond to their questions. She avoids interfering on the discussion because she wants students to be the protagonists and they seem to accept this role:

> The attempt is "this is their moment!" When a group presents its work, the questions are put to the group (...). They are constantly questioning their colleagues and they are always interested...

In this phase, the main concerns of the teacher are to orchestrate student's interventions and to promote the clarification of the emerging mathematical ideas. For her, the discussion goes far beyond the correction of the task and the confrontation of different strategies and solutions constitute enrichment to the students, namely when they discuss wrong solutions:

> I think that if different [solutions] are presented, if they have the ability to have different representations and if I can relate them and establish the connections between them, this is much richer than present only one (...). The idea is also to accept what comes out from them (...) and sometimes we can explore even situations that are not right.

Despite of maintaining a discrete role of orientation, in the observed class the teacher conducted a particular discussion, confronting three of the different solutions presented by the students. Her explanation for such apparent contradictory attitude is based on her vision of the role of the discussion of task:

> I confronted three solutions in the classroom. [The discussion] is a learning moment, so... It cannot be a simple presentation, or a correction, because it is not... it is an opportunity to compare, to think of all the different solutions in different representations... is a goal of the discussion. (...) Is trying to develop their critical sense "in this situation, what is the representation, the solution... what is the strategy that, in fact, helps me more? Because I can have 200 strategies but only one adequate to chose.... "

Systematization of the mathematical learning

After the discussion, which concluded about rules that allow knowing how many stickers are used in a construction with any given number of cubes, Célia adopts a more directive role to conduct a "moment of systematization". She shows the students a table projected on a transparency that she previously prepared, relating the number of cubes and the number of stickers, and challenges them to another question that extends the previous task. The teacher wants to take profit of the discussion of the task to reinforce the importance and power of the rules with letters when exploring situations for any number. She also recalls the inverse operations, establishing connections with student's prior learning:

> After the work in pairs and the collective discussion, there is a part of systematization where I confronted... I started with this table, this number of cubes [52] [and I asked]: "How did you find the number of stickers?" So, we wrote the rule... "And now, knowing the number of stickers, how do we know the number of cubes?" and then came the confrontation of the rules in order to recall the inverse operations....

CONCLUSIONS AND FINAL REMARKS

Table 1 presents, in a synthetic way, the set of actions that Célia performs while teaching and the main intentions that explain or justify her actions.

Table 1: Intentional actions of Célia on her mathematics inquiry-based classroom practice

	Promotion of mathematics learning	**Classroom management**
Introduction of the task	*To guarantee the appropriation of the task by the students:* - Helping students to get familiar with the task's context - Clarifying the interpretation of the task - Setting goals *To promote students' engagement in the task:* - Connecting to students' prior experiences - Challenging students for work	*To organize students' work:* - Setting forms of work organization (groups (pairs) for autonomous work and whole-class for discussion) - Organizing the lesson resources (worksheets, cubes and stickers for each group)
Development of the task	*To support students' autonomous work on the task:* - Making questions and giving clues - Suggesting representations - Focusing productive ideas - Requesting clarifications and justifications - Challenging to deepen and extend the task *To keep the cognitive challenge* - Promoting the students' reasoning - Trying not to validate the mathematical correctness of the students' answers	*To promote the work of the pairs/groups:* - Setting interactions between students - Providing resources for the group *To guarantee the production of materials for the students' presentations:* - Requesting writing records - Providing transparencies *To organize the discussion:* - Identifying and selecting varied solutions (with common errors, less or more complete, and with relevant representations to explore) - Sequencing the selected solutions
Discussion of the task	*To promote the mathematical quality of the students' presentations:* - Asking for clear explanations of the solutions - Asking for justifications of the outcomes and the representations used - Discussing mathematical errors identified in the solutions - Discussing the difference and mathematical efficacy of the presented solutions (tables and rules with letters) *To promote interactions among students in the discussion:* - Encouraging questioning and answering - Encouraging analysis, debate and comparison between solutions	*To create a favorable environment for presentation and discussion*: - Putting an end to the autonomous work of students - Providing the reorganization of the places/space - Promoting an attitude of respect and genuine interest on different presentations *To manage relationships among students:* - Setting the order of presentations - Promoting and managing the participation of students in the discussion

Systematization of the mathematical learning	*To institutionalize ideas and procedures concerning the development of algebraic reasoning raised by the task's exploration:* - Identifying productive representations for obtain generalizations (table) - Recognizing the value of a symbolic rule *To establish connections with prior learning:* - Highlighting links with the targeted mathematical concepts and procedures (idea of rule with letters and concept of reverse operation)	*To create an appropriate environment for the systematization:* - Focusing students at the collective systematization - Promoting students' recognition of the importance of this phase of the lesson for learning *To guarantee written record of the ideas that result from systematization:* - Recording by the teacher (on her transparency previously prepared)

The actions were observed on Célia classroom and the intentions were identified from her final reflection interview. To organize Célia intended actions, we adopted a four phases model for the lesson structure, based on the way she developed her "Cubes and Stickers" lesson: Introduction of the task in interaction with the students; Development of the task by the students; Discussion of the task by the students and herself; and Systematization of the mathematical learning conducted by her. That option allows to distinguish two different purposes of the collective work of the class on Célia practices: the discussion of the task, based on the confrontation and comparison of students strategies, and the put in evidence of the mathematical learning (a concept, an idea, a procedure…) intended by the teacher as a mathematical purpose of the lesson, anchored on the exploration of the mathematical task. It is interesting to note the diversity of roles that the teacher and the students assume on Célia classroom. The practice of a mathematics inquiry-based teaching does not necessarily imply that the students are in the command of the lesson in every moment. It is very clear that Célia assumes a more teacher-centred attitude in the final phase of systematization.

When conducting her lesson, Célia performed actions motivated by two main purposes: Promotion of the mathematics learning of the students and management of the student's work and class as a whole. Most of the actions performed concerning the class management are intended to create better conditions for the mathematics learning. Nevertheless, Célia seemed to have much more predisposition to reflect about the actions directly concerning the mathematics learning of the students. This put an emphasis on the relational dimension of teaching. We want to stress that this characterization of the teacher practices must be seen in the context of her relations to the students: most of the actions of the teacher emerge as response to her interpretation of what are the necessities of the class or of some particular students. She interprets this by listening and observing the students.

This characterization also reinforces the idea of the complexity of developing an inquiry-based mathematics teaching practice. During the lesson, the teacher needs to pay attention to multiple aspects simultaneously and she needs to take management decisions that affect the opportunities of mathematics learning of the students.

Is our expectation that a transversal analysis of different cases of teachers will allow us to elaborate a global framework that provides an authentic and detailed picture of the

mathematics inquiry-based practice of the teachers, and it can serve as a resource for teacher development. We expect to accomplish that objective in the context of the P3M Project, that aims to study mathematics teachers' professional practices and the conditions of their transformations in order to face the challenges of mathematics teaching oriented by the present curricular orientations.

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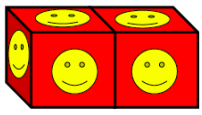
Appendix 1. Cubes with Stickers

Joana is building a game with cubes and stickers. She connects the cubes through one of its faces and forms a queue of cubes. Then she glues a sticker in each of the cube's faces. The figure shows the construction that Joana did with 2 cubes. In that construction she used 10 stickers.



1. Find out how many stickers Joana used in a construction with:
1.1. Three cubes; 1.2. Four cubes; 1.3. Ten cubes; 1.4. Fifty two cubes.
2. Can you find out what is the rule that allows you to know how many stickers Joana used in a construction with any given number of cubes? Explain how you thought.

[1]Adapted from Moss, J., Beaty, R., McNab, S. L., & Eisenband, J. (2005). *The potential of geometric sequences to foster young students' ability to generalize in Mathematics.* http://www.brookings.edu/gs/brown/algebraicreasoning.htm