A Stochastic Model for Managing Tasks of R&D Projects

Joana Fialho[1], Pedro Godinho[2], and João Paulo Costa[3]

[1] Escola Superior de Tecnologia e Gestão de Viseu, Portugal
Instituto de Engenharia de Sistemas e Computadores de Coimbra
(E-mail: jfialho@estv.ipv.pt)
[2] Faculdade de Economia da Universidade de Coimbra, Portugal
Grupo de Estudos Monetrios e Financeiros, Coimbra
(E-mail: pgodinho@fe.uc.pt)
[3] Faculdade de Economia da Universidade de Coimbra, Portugal
Instituto de Engenharia de Sistemas e Computadores de Coimbra
(E-mail: jpaulo@fe.uc.pt)

Abstract. In this paper, we propose a model for managing tasks of R&D projects. We assume that different amounts of resources may be allocated to a task, leading to different costs, and different average execution speeds. The advancement of the task will be stochastic, and the manager may change the allocated amount of resources according to the way the task is progressing. The revenues will depend on the time to the completion of the task, and their expected value will follow a stochastic process. We consider that a strategy for completing the task will consist on a set of rules that define the level of resources to be chosen at each moment, according to the values of several state variables. We discuss the evaluation for completing the task, and we briefly address the problem of finding the optimal strategy.

Keywords: R&D evaluation, real options, stochastic models, simulation, optimal decisions.

1 Introduction

Companies need to find new opportunities of investment in competitive markets. These investments must be evaluated and it is necessary to incorporate management flexibility and uncertainty in the evaluation.
To choose the projects to invest, companies need a tool to evaluate them. The project evaluation process must provide the most adequate decisions as well as the management strategy to be followed. Traditional project evaluation methods, such as the ones based on discounted cash flows, are not adequate because these kind of methods assume a determined and a fixed plan, which does not permit to take into account both uncertainty and flexibility [8]. Plus, traditional methods ignore the irreversibility and the option of delaying an investment [1].
The real options theory is useful to evaluate dynamic and uncertain projects, because it takes into account the necessary actions and decisions to achieve

the maximum value that the project can assume. This theory also allows to analyze business and investment strategies as real options that can be exercised during the project [8].
The model presented in this paper aims to support management of tasks of R&D projects and the basis of its construction was Godinho *et al.* (2007), who proposed a real options model for the analysis of R&D projects from the telecommunications sector.
We assume that different levels of resources can be allocated to a task, which have different costs and different average execution speeds. We also assume that each task needs a certain number of work units to be completed and that these units can be executed with different levels of resources. Thus, this model can be used to define the optimal strategy and proceed to its evaluation, that is, this model allows to determine which is the better level of resources to use at each moment.
The expected revenues follow a stochastic process and depend on the time to complete the task. The concept of instantaneous revenue is used, which represents the present value of the task revenues, assuming that the task was already finished. We assume that this present value grows at a pre-defined rate (*e.g.* inflation consequences), and in certain moments, there can occur jumps in this growth (*e.g.* entry of new competitors on the market). We also assume a penalty in the revenues, that is, the revenues are more penalized as the task takes longer to be completed.
We consider that the time to complete a task is not deterministic, because it is not possible to know the exact time that the task will take, since it belongs to an R&D project, subject to uncertainty. Thus, we assume that the time to complete a task is described by the sum of a deterministic term and a stochastic one. The deterministic term represents the minimum time that the task lasts and the stochastic one represents the uncertainty of the duration of the task. This last term is constructed based on the Poisson distribution. This distribution is the basis of the Huisman, Farzin and Kort work, that use it to model the arrivals of new technologies with or without competition [2], [4], [5], [6]. This work helped to define the random term we incorporated in the time to finish the task.
It was considered that the costs are deterministic *per* unit of time and depend on the level of resources. We also assumed that there might be a cost to switch among different levels of resources, that is, the change from a level to another one may entail a cost.

2 The characterization of the model

The model we propose aims to represent strategies to execute tasks of R&D projects and we may then use it to define the optimal strategy and to perform its evaluation. A task is completed if a certain number of work units is done. These work units can be executed by different levels of resources, which lead to different average times to finish the task and different costs, *per* time unit. The next subsections describe, in detail, how we treat the time to complete a task, the revenues, the costs and the value of the task. With this model, we are able to define the strategies for completing the task.

2.1 The time to complete the task

The time to finish a task uses the exponential and the Poisson distributions. The time to complete a task is uncertain, because of delays that can emerge due to technological difficulties, or others. For the companies which develop this kind of projects, the essential part of the evaluation is the remaining time to finish the task and the level of resources they may use.

The time to finish a task with the level of resources k is defined as T_k. Each task has a deterministic minimum time that depends on the level of resources. This minimum is defined as M_k, where k is the level of resources. Besides this minimum, T_k is a random variable, because it is impossible to know, exactly, how much time remains, due to unpredictable delays or technical difficulties. The definition of T_k will use the Poisson distribution. Let D be the necessary number of work units to complete the task. These units are considered as Poisson arrivals, that is, D is the necessary number of Poisson arrivals to complete the task. The value of D can be defined technically, that is, it can be given *a priori*. The number of work units that each resource level can conclude *per* unit of time is the parameter μ of a Poisson distribution. Thus, we define $t \equiv$"Time between two consecutive Poisson arrivals", that is, t is defined by an exponential distribution with average $1/\mu$. The value of μ can be indirectly elicited through the average times or the work percentage done *per* unit of time.

Suppose a specific level of resources. The time to complete the task is defined by T, and it is immediate that $T = \sum_{i=1}^{D} t_i$, where t_i has the same distribution of t. However, the time T to complete a task has to incorporate a deterministic term M, that is the deterministic minimum time to finish the task. Thus, each term t_i can be written as $t_i = \frac{M}{D} + \widetilde{t}_i$, where $\widetilde{t}_i$ follows an exponential distribution. Replacing t_i in the time T,

$$T = M + \sum_{i=1}^{D} \widetilde{t}_i$$

So, the time to finish the task is composed by a sum of a deterministic term, which represents the minimum time that is necessary to finish the task, with a stochastic term. This later term is defined as the sum of D independent exponential variables with the same parameter.

2.2 The costs

Let C_t be the instantaneous cost. It will be considered that, for each level of resources, the variation of the costs just depends on a rate. This rate can easily be explained by inflation. So, the model for the costs can be defined by

$$dC_t = \rho C_t dt$$

The value of C_t is the solution of the differential equation $\frac{dC}{dt} = \rho C_t$, that is $C_t = C_0 e^{\rho t}$. The value of C_t represents the instantaneous cost, at instant t. We consider that the time is discretized as $\Delta t = 1$. The present value of the period T cost, using this discretization and considering the discount rate r, is given by $C_0 e^{\rho T} e^{-rT}$.

We also assume that other costs, related with the change of level of resources, can occur. Thus, we assume that there are costs for changing the level of resources being used, that is, if there is a switch of level, there is a cost for that. We also assume that these costs are deterministic and depend on the level of resources.

For the model, it is necessary to calculate the present value of the total costs from a certain work unit j until the end of the task and it is defined by $CT(j)$. Its expression can be given by

$$CT(j) = \mathcal{C}(j) + \sum_{a=j+1}^{D} \left(\mathcal{C}(a) + \gamma(a-1, a) e^{(\rho - r)(tt(a) - tt(j))} \right)$$

where $\mathcal{C}(\cdot)$ is the present value of the cost of the respective work unit, $tt(\cdot)$ represents the instant at the beginning of the respective work unit, r is the discount rate and $\gamma(k, h)$ defines the costs to change from the resource level used in work unit k to the resource level used in work unit h.

2.3 The revenues

The revenues do not depend on the level of resources used to undertake the task, but on the time to the completion of the task. For the revenues, we use the concept of instantaneous revenue, that is the present value of revenue assuming that the task is already finish.

We assume that the variation of the revenues depends on a rate (that can be positive or negative) and on a jump process. Thus, let be the model of revenues defined by

$$dR = \rho R dt + R dq$$

where dq represents a Jump process, that is

$$dq = \begin{cases} 0, \text{ with probability } 1 - pdt \\ u, \text{ with probability } pdt \end{cases}$$

with u defined by a uniform distribution,

$$u \sim U(k_1, k_2), \quad -1 \leq k_1 < k_2 \leq 1$$

The parameter ρ represents the increasing or decreasing rate of the revenues, in each lapse dt and its value is included in $[-1, 1]$.
Also for the revenues, the time is discretized with $\Delta t = 1$ and it is assumed that in a lapse of time that is not infinitesimal, more than one jump may take place. So, in moment $t + 1$ we have

$$R_{t+1} = R_t + \rho R_t + R_t \Delta q$$

where

$$\Delta q = \sum_{i=1}^{v} u_i$$

with $u_i \sim U(k_1, k_2)$, and v defined by a Poisson distribution with parameter p.
For the first period revenues R_1, it is necessary to know the initial value R_0, which is an input for the model. Notice that the value for R_{t+1} is the value for what we defined as the instantaneous revenue at moment $t + 1$, that is, R_{t+1} is the present value of revenue assuming that the task is already finished at $t + 1$.
Assuming that the task is finished at instant T, R_T is the task revenue, as follows from the description above. However, we also assumed a penalty in the revenues, that is, the revenues are more penalized as longer the task is. This feature is adequate, because of the competition of the market, that is, if a competitor is able to introduce a similar product in the market earlier, the revenues might be lower. So, we assume that the earlier the product is launched in the market, the bigger are the revenues obtained. The penalty mentioned can be expressed by a function $f(t)$, where t expresses the time. This function is positive, continuous, decreasing, and it takes values from the interval $[0, 1]$. Thus, assuming this feature, the final expected revenue is $R_T \times f(T)$.

2.4 The value of the task

For the model, it is necessary to calculate the task value, for each work unit j, $j = 1, ..., D$. The task value includes the present value of the expected revenue at the end of the task and the present value of the total costs from the work unit j until the last one D. Thus, assuming that the time to complete the task is T, the value of the task, at the beginning of the work unit j is $Val(j)$, and it is determined as follows:

$$Val(j) = R_T \times f(T) \times e^{-r(T-tt(j))} - CT(j)$$

where $tt(j)$ is the time elapsed until the beginning of the work unit j, $CT(j)$ is the present value of the total costs from the work unit j until the last one D, and r is the discount rate.

3 Guidelines for the evaluation process

It is assumed that each task under consideration needs a number D of work units to be finished, and that each work unit can be executed by different levels of resources. As it was previously described, each resource level has different average times to execute the work units, as well as different costs *per* unit of time. With the descriptions above, different strategies are constructed using one or more levels of resources. These strategies are initially simulated through a large number of paths. These paths allow to execute the evaluation process, that is based on the simulation method of Least Squares Monte Carlo [7]. The objective of this process is to build, in a regressive way, regression equations that explain the task value as a function of different state variables, which are the time elapsed, the instantaneous revenue and the number of work units already finished. These state variables were chosen, because their values are known at each moment. It is considered that the value of the task, at each instant, is the present value of the difference between the revenue at the end of the task and the costs incurred from that instant to the end of the task.

The process begins in the last work unit. Assuming the paths simulated initially, each level of resources is considered to complete the last work unit. For each level of resources, a regression equation is built that explains, in the last work unit, the task value as a function of the time elapsed until the beginning of that last unit and of the instantaneous revenue observed in the beginning of that unit. For the earlier work units, the process is quite similar: it is considered that the work unit under consideration is executed by each of the different levels of resources. For each level of resources, in that work unit, a regression equation is constructed that explains the value of the task as a function of the time elapsed and of the instantaneous revenue. This construction needs the task values calculated from the simulation/paths. Thus, for the work unit at issue, the level of resources considered is used. Then, the regression equations already determined are used to construct the best strategy, since the following unit until the last one, that is, for the following units, the best option, for each of the following work units, is the level of resources that leads to a higher value in the respective equation. With the optimal strategy, the task value is calculated and the regression equation is built. This process proceeds, in a regressive way, until the first unit.

So, in the end of the process of construction of the regression equations, the optimal strategy is built, for each path simulated initially. This optimal strategy allows us to calculate the value of the task, for each unit, since the first until the last one.

4 Conclusion

This work presents a model for managing tasks of R&D projects, which can be executed by different resources levels. These resource levels have different average speeds to complete the task as well as different costs. We discussed the ways to incorporate, in the model, the time to complete a task, the revenues, the costs, and the task value. This task value is used for the evaluation process.
For future work, the model presented can be the basis for the evaluation of an R&D project defined as a network of tasks, that is, we intend to develop a model that evaluates multiple tasks that are connected among them and that may have dependence among them.

References

1. A. Dixit, R. Pindyck (1994), *Investment under uncetainty.* Princeton University Press, New Jersey
2. Y.H. Farzin, K.J.M. Huisman, P.M. Kort (1998), *Optimal timing of technology adoption*, Journal of Economic Dynamics and Control, 22, 779-799.
3. P. Godinho, J. Regalado, R. Afonso (2007), *A Model for the Application of Real Options Analysis to R&D Projects in the Telecommunications Sector*, Global Business And Economics Anthology, vol II, 412
4. K.J.M. Huisman, P.M. Kort (2002), Strategic technology investment under uncertainty, OR Spectrum, 24, 79-98
5. K.J.M. Huisman, P.M. Kort (2003), *Strategic investment in technological innovations*, European Journal os Operational Research, 144, 209-223.
6. K.J.M. Huisman, P.M. Kort (2004), *Strategic technology adoption taking into account future technological improvements: A real options approach*, European Journal os Operational Research, 159, 705-728.
7. F. Longstaff, E. Schwartz (2001), *Valuing American Options by Simulation: A Simple Least-Square Approach*, The Review of Financial Studies 14, 1113-1147.
8. K.T. Yeo, Fasheng Qiu (2003), *The value of management flexibility - a real option approach to investment evaluation*, International Journal of Project Management, 21, 243-250