

such as number theory, set theory, measure theory, combinatorial theory, probability theory, graph theory, and cryptography.

Examples from the history of mathematics show that the IEP has been used in different areas, although it has not always been explicitly formulated. Classical texts on the history of mathematics emphasize examples of the use of the IEP in problems of games of chance studied in the early eighteenth century, presenting detailed explanations of the mathematical contexts. Notable are the contributions of Pierre Rémond de Montmort (1678–1719), in his *Essay d'analyse sur les jeux de hazard* (1708, with a second edition in 1713), of Abraham De Moivre (1667–1754), in *De Mensura Sortis* (1711–12), and of Nicolaus Bernoulli (1687–1759), in his correspondence with Montmort from 1711 to 1713, published in the second edition of Montmort's *Essay*. These cases will be referred to in Section 2. We will give a summarized description of the problems that prompted the uses of the IEP, providing references for further details.

Other examples of the use of the IEP are covered in the existing literature, namely the contributions of the French mathematicians Pierre Simon de Laplace (1749–1827) and Henri Poincaré (1854–1912), the mathematician Daniel Augusto da Silva (1814–1878) – one of the most important Portuguese mathematicians of the nineteenth century – and the British mathematician Jean Joseph Sylvester (1814–1897), in 1812, 1896, 1854, and 1883, respectively. However, relevant historiographic considerations are lacking, namely, the context in which the principle is used, i.e., the mathematical problem that originated it, and whether or not, in each case, a formulation of the principle is proposed by the author.<sup>2</sup>

In this paper, our focus will be on the IEP formulations proposed in the nineteenth century. Besides da Silva's (1854), Sylvester's (1883) and Poincaré's (1896) proposals, we will also address the contribution of Adrien-Marie Legendre (1752–1833) in 1808. This last contribution was the starting point of Sylvester's work on the IEP. In Section 3, we will present those contributions. Besides providing the corresponding historical frameworks and original mathematical formulations of each case, we will refer to its versatility, namely its application to problems with a wider scope than the initial one. The awareness of that potential by each of the authors will also be stressed. Concerning our historiographical options, we further emphasize that we are not concerned to give a general historical perspective about the ways the IEP developed. We also consider that questions such as 'who was the first mathematician to ...?', while they have a genuine historical interest when priority disputes are studied, are 'often close to meaninglessness' (Grattan-Guinness 2004, 169) when the notion in question is a basic one like the IEP. We will show the context of the specific problems that gave rise to each one of the different nineteenth-century formulations of the IEP and the concepts, as well as the notations, used by each author in their presentations. Furthermore, we will

<sup>2</sup>In his *Théorie analytique des probabilités* (1812), Laplace uses the IEP to solve a probability problem, but he does not write a formal statement of the principle (Laplace 1812, 253–261). We have identified a brief indication of this in (Takács 1967, 104, 106). In his *Calcul des probabilités* (1896), Poincaré determines the probability that at least one event occurs among a collection of  $n$  events. The solution of this problem is known as the general probability theorem or the probabilistic inclusion-exclusion principle. A brief note of this fact can be found in (Takács 1967, 103). The references to da Silva and Sylvester appear in introductory monographs to the study of combinatorics or discrete mathematics, also with very brief considerations. As examples, see (Ryser 1963, 19; van Lint and Wilson 2001, 96; Erickson 1996, 107; Mazur 2010, 94; and Gallier 2011, 229–236).