TEACHERS' CONCEPTIONS AND PRACTICES ON THE HISTORY OF MATHEMATICS IN TEACHING AT 5TH AND 6TH GRADES IN PORTUGAL

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Abstract

This paper presents some results of the project (H)ISTO é MATEMÁTICA — História da Matemática no Ensino da Matemática (History of Mathematics in teaching), a research project that has as one of its main goals: to describe the initial and continuous training of Portuguese teachers in History of Mathematics (HM), as well as their conceptions and practices on the use of HM in teaching. In particular, we present the results of applying an online questionnaire to a group of 259 Mathematics teachers from the 2nd Cycle of Basic Education in Portugal (5th and 6th grades). With this survey our purpose is precisely to characterize teachers': (i) training in HM; and (ii) conceptions and practices about the utilization of HM in the classroom. Notice that this study intends to respond, among others, the lack of studies on this subject, particularly at the level of the first years. We use quantitative data analysis methods, namely descriptive statistics. The main conclusions of this study are: 1) most teachers considered the training in HM obtained in their higher education as non-existent or reduced; 2) the overwhelming majority categorized as non-existent or reduced their continuous education in HM; 3) most teachers reported using HM as a didactic resource in Mathematics class (many of them in a regular basis); 4) teachers resorted more frequently to textbooks and favoured the use of HM in the introduction to new mathematical subject; 5) regarding to the potential of using HM in teaching, teachers evaluated the didactic potential of HM very positively; the biggest constraints were related to the extension of the official curricula, the scarcity of support materials and the difficulty of evaluation.

Keywords: History of Mathematics, Mathematics teaching (5th-6th grades), teacher training, teachers' practices, teachers' conceptions.

1 FRAMEWORK AND OBJECTIVES

The field of history of mathematics in mathematics education is a domain where the potentialities of using History of Mathematics (HM) are analysed to improve the teaching of Mathematics in all levels. This field began more than half century ago (see, for instance [1]) and was formalized inside the *International Commission on Mathematical Instruction* with the creation of a thematic affiliated organization (*HPM: International Study Group on the Relations between History and Pedagogy of Mathematics*) in 1976. In fact, since the early years, there was the conviction that using HM in the classroom could be pedagogically useful but also could give a contribution to humanize the discipline ([2] and [3]).

This field is currently very active in the search to improve teaching through HM as can be seen, for example, in the project «Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources» [4]. In [5] is described a detailed history of the field and many current examples of the utilization of the HM in teaching (from United States, Brazil and Denmark). For the potentialities of using the HM in the Portuguese educational system, see, for instance, [6], where is analysed which topics of the HM could be useful in teachers' training in Portugal. Notice that, at least since the late 20th century, Portugal was aware of the latest developments in this field; for example, in 1997, the Portuguese Association of Mathematics Teachers, in the context of their Working Group on History and Teaching of Mathematics, published the translation of texts of recognized authors such as Dirk Struik, John Fauvel and Frank Swetz [7]. Short after, in 2001, a long paper [8] was published in one of the journals of this association

presenting the “benefits from integrating History of Mathematics into teaching (notice that the article was written in English, but one of the two authors was a Portuguese researcher). It is also noteworthy that Portugal hosted, in 1996, in Braga, the *Second European Summer University on the History and Epistemology in Mathematics Education*, an important event for the international HPM community [9].

One of the first studies about teacher’s conceptions on HM was done in 2004 in Hong Kong. This investigation was managed by Man Keung Siu [10] and included the answers of 608 participants. This survey concluded that there was a decalage about the perception of the HM utility and the effective use of HM: “the value of history of mathematics is highly regarded by school teachers, but the degree of initiative on actually using history of mathematics in the classroom is very low!” (p. 270). Another important conclusion of Siu was that teacher training in HM could change the situation: “‘preaching the gospel’ significantly enhances both the awareness and the initiative to use history of mathematics in the classroom” (p. 270). The main reasons appointed by teachers to not use HM were the following: “I have no time for it in class!” (67% of the participants agreed with this statement), “there is a lack of resource material on it!” (64%) and “there is a lack of teacher training in it!” (83%) (p. 271).

Another survey to mathematics teachers, this time in France with 646 participants, was presented very recently and produced similar results. This study was presented by Marc Moyon [11] and concluded that 84% of the teachers used HM “as part of the introduction of a pedagogical sequence, by means of an anecdote immersing the pupils in a historical context” (12:30) and that 71% of the teachers are interested in HM and would like to introduce it in their teaching (10:20). Notice that this study appears in the context of the reform of the French high school education system, in 2019, where is stated, for instance, that the problems proposed in classroom can be taken from HM and that it is useful to provide historical contextualization of the mathematical concepts to students (2:50). Notice also that several French researchers, in 2018, presented ten experiments [12] which introduce historical perspectives into mathematics classrooms for 11 to 18-year-olds.

In Portugal there wasn´t, until now, any study that characterize the conceptions and practices of the Portuguese Mathematics teachers on the use of HM in Mathematics teaching [13]. Currently, the authors of this paper have a research project called (H)ISTO é MATEMÁTICA that intends to fill that gap. Another flaw that this project intends to address is the lack of studies that allow to characterize the HM that is presented to Portuguese mathematics teachers in their initial and continuous training. In the following sections we present the results of a survey, within the scope of that project, applied to a group of 259 mathematics teachers from the 2nd Cycle of Basic Education in Portugal (5th and 6th grades, generally, with students between 10 and 12 years old).

2 METHODOLOGY

A quantitative approach was adopted in this study and the collected data was obtained from an online questionnaire. The participants in the study are 259 Portuguese Mathematics teachers from the 2nd Cycle of Basic Education of Mathematics and Science (5th and 6th grades).

Most participants are female (76%), with an average age of 49 years old and 47 years of working service (on average). 30% of teachers teach or have taught in the 3rd Cycle of Basic Education (7th to 9th grades) and 26% in Secondary Education (10th to 12th grades). The majority of teachers (71%) have an undergraduate degree (*licenciatura*) and 17% have master’s degree (mostly in Mathematics Education).

This study aimed to characterise (i) their training in HM; and (ii) their conceptions and practices about the use of HM.

The questionnaire is organized in three sections related to the dimensions of the study: teachers' training in HM, teachers' conceptions on the use of HM and teachers' practices in using HM in their teaching practice. In each section there were closed-response questions, with a four-level agreement scale from 1 (minimum agreement) to 4 (maximum agreement), and open-response questions.

Data analysis uses descriptive statistical analysis techniques (frequencies and measures of descriptive statistics). The coefficient of variation (CV) was used to evaluate the representativity of the mean-value (the mean-value will be the more representative the lower the CV value; for CV>50% the mean-value is not considered representative) ([14]).

Calculations were performed with Ms Excel software (15.32 version).

The following categories of analysis where considered (Table 1).

Table 1 - Subjects and analysis categories

Subjects	***Categories***
Training in HM	• Initial training • Continuous Training
Conceptions on the use of HM for teaching	• Potential for the use of HM • Constraints to the use of HM (student, training, curriculum guidelines and resources, nature of the subject)
Practices of using HM for teaching	• Ways in which HM can be used in the classroom • HM resources for the classroom • Impact of the use of HM on student learning

3 RESULTS

The results of this study are organized in the following sections: (i) training in History of Mathematics; (ii) conceptions of teachers on the History of Mathematics; and (iii) practices of teachers on the use of History of Mathematics in Mathematics teaching.

3.1 Training in History of Mathematics

In this section, we describe the initial training and continuous training in HM of the inquired teachers.

Most teachers considered the training in HM obtained in their higher education as *non-existent or reduced* (62%). 12% of them considered it as *solid* (obtained, for example, in subjects of HM).

Invited to specify that training, only 62 of the teachers answered (27%). Most of them (60%) mentioned that their training in HM was obtained in specific subjects of HM, while the rest referred that it was integrated in subjects of different areas (Didactics of Mathematics, Calculus, Statistics, Theory of Numbers).

As for continuing training in HM, 51% of the inquired teachers consider it as *non-existent*, 44% classifies it as *reduced* and only 5% consider it *frequent*.

Invited to specify that continuous training, only a minority of them answered (10%). Results are equally distributed between the attendance of conferences, long-term training courses and meetings specific to teachers of Mathematics (namely *ProfMat*).

3.2 Conceptions of teachers on the use of History of Mathematics in Mathematics teaching

Conceptions of teachers on the use of HM in Mathematics teaching were studied in two domains: potentialities and constraints.

Table 2 presents the results to the question: "Appreciate the potential of using History of Mathematics in the Mathematics classroom". Teachers appreciated very positively the potential of using HM (80% to 90% levels 3/4; averages between 3.1 and 3.4 points, representative of the data set). The illustration of the usefulness and importance of Mathematics stands out (90% levels 3/4; average 3.4).

Table 2. Teachers' average agreement on the potentialities of using HM ($\bar{x}$) and coefficient of variation (CV)

Item	$\bar{x}$	*CV (%)*
It favors the demystification of mathematics as a finished product, showing that doubt and error are part of human (mathematical) activity.	3.2	23
Allows to illustrate the usefulness and importance of Mathematics.	3.4	20
Allows to illustrate relationships between different mathematical domains.	3.2	22
Allows you to illustrate relationships between mathematics and other disciplines.	3.3	21
Allows a different approach to Mathematics than usual.	3.2	24
It enables the development of skills beyond mathematical knowledge, such as documenting, analyzing and discussing mathematical subjects.	3.1	24
It encourages students to be motivated to learn Mathematics.	3.1	26

Teachers were also inquired on the constrains they recognize in using HM in the Mathematics classroom — table 3 shows the results. The constraints that received higher levels of agreement are included in the dimension *curricular guidelines and resources* (average 2.9). The item *extension of the program* (90% levels 3/4; average 3.1) stands out, followed by the *integration of HM in the evaluation* and by *the scarcity of support materials*. The constraints with lower levels of agreement have to do with the *nature of the discipline* (average 1.8, with small representativity).

Table 3. Teachers' average agreement on the constrains of using HM ($\bar{x}$,$\bar{x}_i$) and coefficient of variation (CV)

	Item	$\bar{x}_i$	$\bar{x}$	*CV (%)*
Students	Students don't have enough knowledge to understand and appreciate contents of the History of Mathematics.	2.3	2.5	36
	Many students do not appreciate History so, in general, they will not enjoy History of Mathematics.		2.1	42
Teachers' training	The knowledge I have gained in my training about the History of Mathematics limits its use in Mathematics class.	2.5	2.5	40
Curricular guidelines and resources	Materials that support this type of approach are scarce, or non-existent.	2.9	2.8	33
	The course syllabus is extensive and makes it difficult, or even prevents, the inclusion of this didactic approach.		3.1	31
	The integration of the History of Mathematics in the assessment of the subject is hampered by the lack of curricular guidelines.		2.8	34
Nature of the discipline	Episodes from the History of Mathematics make mathematical topics more complex.	1.8	1.8	47
	Mathematics progresses towards simplifying difficult problems, so there is no reason to worry about the past.		1.6	52

3.3 Practices of teachers on the use of History of Mathematics in the Mathematics classroom

This section includes the analysis of teachers' practices on the use of History of Mathematics in their classes.

A large majority of teachers (80%) reported using or having already used HM as a didactic resource. 13% do it/ did it a *few times* and 62% do it/did it *sometimes.*

Table 4 shows the results on teachers' ways of using HM. The most frequent situations appointed by teachers were the *introduction to mathematical content* (90% levels 3/4; average 3.4). As less frequent teachers mentioned the use of HM *focused on teacher's presentation* and *focused on the resolution of mathematical tasks* (30% levels 3/4; averages 2.1). All items were evaluated positively (except for the latter, the mean values varied between 2.5 and 3.4 and are representative).

Table 4. Teachers' average agreement on ways of using HM ($\bar{x}$) and coefficient of variation (CV)

Item	$\bar{x}$	*CV (%)*
Introduction to mathematical content.	3.4	21
Alluding to the history of mathematical symbology.	3.1	25
In the development of a mathematical content.	2.5	35
Focused on teachers' presentation.	2.1	45
Focused on the resolution of mathematical tasks.	2.1	43
Proposing projects extra-class research or projects	2.6	40

Regarding the use of resources in teaching practices with HM, teachers' levels of agreement are summarized in table 5. There is a clear predominance of the *scholar textbook* (70% levels 3/4; average 2.9), followed by *texts/videos/websites* (more than 60% levels 3/4; average 2.8). The least used resources were *teaching material elaborated by the teacher*; *ancient instruments* and *primary sources* (averages 1.7, 1.8 and 2.0), where the averages of the last two items are not representative.

Table 5. Teachers' average agreement on resources mobilized in the use of HM ($\bar{x}$) and coefficient of variation (CV)

Item	$\bar{x}$	*CV (%)*
Primary sources (excerpts from original mathematical documents).	2.0	53
Texts/videos/websites with narratives/interpretations/historical reconstructions.	2.8	34
Teaching material related to the History of Mathematics.	2.4	41
Teaching material related to the History of Mathematics prepared by me.	1.7	52
Scholar textbooks.	2.9	30
Mathematicians' biographies.	2.4	41
Problems with mathematical context.	2.3	42
Ancient games.	2.3	43
Ancient instruments (mechanical/mathematical).	1.8	48

Invited to describe one of their teaching practices using HM, the overwhelming majority answered (90%). The examples reveal a strong connection with scholar textbooks (Pythagoras theorem, sieve of Eratosthenes, numeration systems, Pi number, Pedro Nunes' method of solving equations, short biographies of mathematicians, …).

Regarding the recognition of the impact of using HM on student's learning (table 6), teachers do not differ much in the assessment of the different items, with positive results (levels 3/4 between 55% and 80%, averages between 2.6 and 3.0 and representative). The items with the highest levels of agreement are *facilitating the establishment of connections between Mathematics and reality* and *increased motivation for solving the proposed tasks* (more than 70% at levels 3/4 and averages 3.0 and 2.9). *Facilitating the understanding of mathematical content* was the less punctuated item (average 2.6 and about 50% at levels 3/4).

Table 6. Teachers' average agreement on the impact of using HM on students' learning

Item	$\bar{x}$	*CV (%)*
It facilitated the understanding of mathematical content.	2.6	33
It allowed the development of transversal skills such as mathematical communication, problem solving and mathematical reasoning.	2.7	33
Motivation for solving the proposed tasks increased.	2.9	30
The establishment of connections between Mathematics and reality was facilitated.	3.0	27
The establishment of connections between Mathematics and other areas of knowledge was facilitated.	2.8	29
Students enjoyed more Mathematics.	2.9	29

4 DISCUSSION AND CONCLUSIONS

Most of the inquired teachers considered that their initial training and their continuous training in HM is reduced or non-existent; this fact could be a potential obstacle to a more effective use of HM in teaching. However, most teachers reported using or having already used HM as a didactic resource in Mathematics classroom (and many of them do it regularly). In their practices, they resorted more frequently to scholar textbooks and favoured the use of HM in the introduction to new mathematical content, practices that are in accordance with international studies ([4], [5]). Notice that, teachers evaluated the didactic potential of HM very positively, but they also appoint major constraints for using HM more often: the extension of the official curricula, the scarcity of support materials for HM and the difficulty of integrating HM in the students' evaluation. Some of these constraints were also referred in [10], in particular the ones related with the time needed to use HM in classroom. Our results are in accordance with Moyon [11] in France, and points that there is receptivity for more teacher training in HM (initial and/or continuous) as well as for HM pedagogical materials specifically prepared to be used in the classroom context.

The results obtained in this study are aligned with international studies on the use of the HM in teaching. This study allows us to conclude that Portuguese Mathematics teachers of the 5th and 6th grades recognize the need for more training in HM, which implies that they would be receptive to training in this area. It was also found that teachers feel a lack of specific teaching material for the use of HM in their classes, supporting them in preparing their practices and also in new ways to make students' evaluation when using HM. We identified implications for the initial and continuous teachers' education, namely, the need to create teaching materials for the use of HM in teaching and to disseminate them to teachers through workshops that train them on how to implement and evaluate HM in their practices. Notice that the majority of teachers evaluated positively the potential of HM in the classroom, which means that it's not a priority to implement actions/activities to work on the mathematics teachers' conceptions.

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