PORTUGUESE TEACHERS' CONCEPTIONS AND PRACTICES ON THE HISTORY OF MATHEMATICS IN TEACHING (7TH TO 12TH GRADES)

Ana P. MARTINS, Hélder PINTO, Helena GOMES, Luís MENEZES
ESE Viseu and CIUHCT; Piaget Institute, RECI and CIDMA;
ESE Viseu and CIDMA; ESE Viseu and CI&DETS
anapatmartins@gmail.com, helder.pinto@ipiaget.pt,
hgomes@esev.ipv.pt, menezes@esev.ipv.pt

ABSTRACT

This study, based on what are the international recommendations for the introduction of the History of Mathematics (HM) in the teaching of this discipline, seeks to describe the Portuguese reality. Thus, the study aims to know: (i) teachers' training in HM; and (ii) their conceptions and practices about the use of HM in the classroom. An online questionnaire was answered by a group of 432 Portuguese Mathematics teachers of secondary education (7th-12th grades). Approximately half of the teachers considered their training in HM obtained in their higher education as frequent or solid. Almost all categorized as non-existent or reduced their continuous education in HM. Regarding the teachers' practices, an overwhelming majority reports using HM as a didactic resource in Mathematics class and only about a third do it rarely. Teachers resorted more frequently to textbooks and problems with a historical context and favored the use of HM in the introduction to mathematical content. As for the potential of using HM in teaching, teachers evaluated the didactic potential of HM very positively. The biggest constraints were related to the extension of the official curricula, the difficulty of evaluation and the scarcity of support materials.

1 Introduction

The topic of using HM as a resource for teaching Mathematics is already quite old in research (Clark, 2019; Martins et al., 2021a). Some of these studies focus on the characterization of what teachers think and do with HM in Mathematics classes. Other studies are dedicated to developing and evaluating didactic proposals that consistently incorporate HM in Mathematics classes.

Although in Portugal the interest in HM comes from the last two decades of the 20th century (GTHEM, 1997), knowledge about the reality of schools, mathematics teachers' perspectives and teachers' training are limited. Therefore, within the scope of the project "(H)ISTO é Matemática: História da Matemática no ensino da Matemática" (History of Mathematics in teaching), we developed this study with the aim of knowing: (i) secondary (7th-12th grades)

teachers' training in HM; and (ii) their conceptions and practices about the use of HM in the classroom. Notice that the subject of HM it's not mandatory in the initial training of these teachers, although some of them may have had topics or some curricular units about it.

2 State of the art

The field of HM in mathematics education is a domain where the potentialities of using HM are analysed to improve the teaching of Mathematics at all education levels. This field was formalized inside the ICMI with the creation of a thematic affiliated organization in 1976 (HPM). This field is still active today; for instance, in Clark (2019), it is possible to find current examples from the utilization of HM in teaching, in Brazil, Denmark and USA.

Portugal, since the late 20th century, was aware of the latest developments in this field: Portugal hosted, in 1996, in Braga, the *ESU2*, an important event for the HPM community; the Portuguese Association of Mathematics Teachers published in 1997 the translation of texts of relevant authors such as D. Struik, J. Fauvel and F. Swetz (GTHEM, 1997); a long paper was published in 2001 in one of the journals of this association presenting the "benefits from integrating History of Mathematics into teaching" (Ferreira & Rich, 2001).

One of the first studies about teachers' conceptions on HM was done, in 2004, in Hong Kong. This investigation was managed by M.-K. Siu and included 608 participants, with the main conclusion: "the value of history of mathematics is highly regarded by schoolteachers, but the degree of initiative on actually using history of mathematics in the classroom is very low!" (Siu, 2007). Siu appointed the main reasons for the teachers do not use HM in the classroom: "I have no time for it in class!" (67% agreed with this statement), "there is a lack of resource material on it!" (64%) and "there is a lack of teacher training in it!" (83%). More recently, a similar study was conducted in France by M. Moyon (646 participants) with similar conclusions: for instance, "71% of the teachers are interested in HM and would like to introduce it in their teaching", but many "do not know how to do it" (Moyon, 2021).

The present work is part of the project "(H)ISTO é Matemática: History of Mathematics in teaching; in (Martins *et al*, 2021a, 2021b, 2021c) and (Costa *et al*, 2021) it's possible to find more results of this project.

3 Methodology

An online questionnaire was answered by 432 Portuguese Mathematics teachers of secondary education (7^{th}-12^{th} grades; generally, students 12-18 years old).

The teachers surveyed were mostly female (78%), with an average age of 52 years old, an average service time of 24 years. Nearly one quarter have a master/doctor degree, most in Mathematics Education.

The survey was organized into three sections according to the themes and categories of analysis defined (Table 1):

Themes	Categories
Training in HM *(2 questions)*	• Initial training • Continuous Training
Conceptions on the use of HM for teaching *(2 questions)*	• Potential for the use of HM • Constraints to the use of HM (student, training, curriculum guidelines and resources, nature of the subject)
Practices of using HM for teaching *(7 questions)*	• Ways in which HM can be used in the classroom • HM resources for the classroom • Impact of the use of HM on student learning

Table 1. Themes and categoris of analysis.

For the answers, we adopted a four-level agreement scale, 1 being the minimum and 4 the maximum levels of agreement. Quantitative data analysis methods were used, namely descriptive statistics.

4 Results

The study's findings are categorized into three categories: (i) training in HM; (ii) teachers' conceptions on the HM; and (iii) teachers' practices related to the use of HM in Mathematics teaching.

4.1. Training in History of Mathematics

In this section, we talk about the initial and continuous training in HM of the interviewed teachers.

Most of them (49%) considered their higher education's HM training as non-existent or minimal and 25% said it was solid (obtained, for example, in

subjects of HM). 50% of the teachers specified their training: most of them (95%) referred it was obtained in specific subjects of HM. Continuous training in HM was said to be non-existent by 46% of the teachers, reduced by 50%, and frequent by 4%. Only a minority (20%) responded to the request for examples and most of them specified short-time training courses.

4.2. Conceptions of teachers on the use of History of Mathematics in Mathematics teaching

In this section, we describe the conceptions on the use of HM in Mathematics teaching of the 432 inquired teachers. Those conceptions were studied in two categories: potential of the use of HM and constraints to the use of HM.

Teachers appreciated in a very evident way the potential of using HM in Mathematics teaching. The survey results showed that, in all items, the levels of agreement 3 or 4 points appear between 75.2% and 88.6% of answers and the average of the scores (arithmetic mean) is high (between 3 and 3.3). Also, the answers highlight the understanding that HM allows to illustrate the usefulness and importance of Mathematics (with 88.6% of answers at levels 3 and 4), allows also to illustrate relationships between different mathematical domains (84.7% in the same levels) and it favours the demystification of mathematics as a finished product, showing that doubt and error are part of the human (mathematical) activity (82.47% in the same levels). The item with the least frequency of response at level 4 (and at levels 3 and 4 together) is the one relative to which HM enables the development of skills beyond mathematical knowledge, such as documenting, analysing, and discussing mathematical subjects (only with 27.5% in level 4).

On the results on the constraints to the use of HM, teachers' opinions were more dispersed since the CV is higher than in the previous data (CV is a measure of dispersion that allows evaluating the representativeness of the mean by the proportion of the standard deviation in relation to the mean value; if CV > 50%, the mean is not representative of the data set and the smaller the CV, the greater the representativeness of the mean). Still, we consider that the mean is representative in all cases. Teachers recognize as major constraints aspects related to the curriculum guidelines, namely the fact that the course syllabus is extensive and makes it difficult, or even prevents, the inclusion of this didactic approach (94.5% in levels 3 or 4). They also identify the

difficulty of evaluation because the integration of the HM in the assessment is not hampered by curricular guidelines (average of 2.9). The scarcity of support materials is also chosen (average of 2.8 and 70.6% in levels 3 and 4). The nature of the discipline is the aspect least identified as a constraint (the lowest average, 1.6).

4.3. Practices of teachers on the use of History of Mathematics in the Mathematics classroom

Most teachers (82%) said they use/already used HM as a didactic resource. 11% do/has done it several times and 60% do/have done it occasionally.

On the results of teachers' ways of using HM, the most frequent use was the introduction of mathematical content (87% levels 3 or 4, average 3.4). Also rated positively was the allusion to the history of mathematical symbology (average 2.9). The less rated items were teachers' presentations and resolution of mathematical tasks (31% levels 3 or 4, averages 2.1). (All the averages respecting the topic ways of using HM are representative.)

Teachers' levels of agreement about the use of resources in teaching practices with HM shows that scholar textbooks predominate (66% levels 3 or 4, average 2.8), followed by problems of mathematical context (more than 62% levels 3 or 4, average 2.7), text/video/websites (more than 57% levels 3 or 4, average 2.6). The least used resources were primary sources; teaching materials developed by teachers and ancient instruments (averages 2.0, 1.8, and 1.8; not representative averages).

An overwhelming majority (97%) responded when asked to describe one of their teaching methods with HM. Such examples show strong links with scholar textbooks: Pythagorean theorem, construction of sets of numbers, Pi number, Thales' theorem and mathematical biographical notes.

Regarding the perceived effects of HM use on students learning, teachers did not differ significantly in their ratings (CV values are low), with positive results (53% to 72% levels 3 or 4, averages between 2.5 and 2.9 and representative). The items with the highest levels of agreement were facilitating the establishment of connections between Mathematics and reality; increased motivation for solving the proposed tasks and increased enjoyment of students (over 60% at levels 3 or 4, averages 2.9, 2.8 and 2.8). The lowest score was

facilitating the understanding of mathematical content (average 2.5 and near 50% levels 3 or 4).

5. Discussion and conclusions

The results presented for the Portuguese teachers of secondary education (7th-12th grades) are very similar to the results of studies for other countries (Siu, 2007; Moyon, 2021): there is a gap between the conceptions (the potential of HM is highly regarded by teachers) and practice (the effective use of HM in classroom is very intermittent). The study also allows us to conclude that half of the teachers said that their initial training in HM is solid; the majority reported that their continuous training in HM is reduced or non-existent. Regarding their conceptions on the use of HM in Mathematics teaching, teachers recognize the potential of using the HM to learn Mathematics, particularly because it allows to illustrate the usefulness and importance of Mathematics, it also favours the demystification of mathematics as a finished product, and allows to illustrate relationships between mathematics and other disciplines. The biggest constraints are related to official curricula, the difficulty of evaluation and the scarcity of support material. Most teachers reported using or having used HM as a teaching resource in mathematics class (and many do this regularly).

Teachers used more often scholar textbooks, as well as problems of mathematical context and text/video/websites in their practices and preferred the use of HM in the introduction to new mathematical content. Teachers evaluated the impact of using HM on students' learning very positively. There is an enormous consensus among teachers from the 7th up to the 12th grades on the need for more training in HM.

Finally, it should be highlighted that these results are aligned with the conclusions presented in (Martins, 2021c), where a similar survey was applied to 259 Portuguese teachers of the 2nd Cycle of Basic Education (5th-6th grades).

REFERENCES

Clark, K. (2019). History and Pedagogy of mathematics in mathematics education: History of the field, the potential of current examples, and directions for the future. In U. T. Jankvist, M. Heuvel-Panhuizen & M. Veldhuis (Eds.), *Eleventh Congress of the European Society for Research in Mathematics Education* (pp.

29-55). Utrecht University.

Costa, C., Menezes, L., Ribeiro, A., Pinto, H., Martins, A. P., Clain, T., Gomes, H. & Aires, A. P. (2021). Perspectives of Future Teachers of Early Years from two Portuguese Institutions on the History of Mathematics. In L. Chova, A. Martinez, I. Torres (Orgs.), *Proceedings of ICERI2021 Conference* (pp. 8222-8230). IATED.

Ferreira, R. and Rich, B. (2001). Integrating history of mathematics into the mathematics classroom, *Quadrante*, *10*(2), 67–96.

GTHEM (1997). *Relevância da História no Ensino da Matemática*. APM.

Martins, A. P., Clain, T. C., Costa, C., Pinto, H., Ribeiro, A., Gomes, H. & Menezes, L. (2021a). Conceções e Práticas de Professores sobre a História no Ensino da Matemática num Contexto de Formação, *Millenium*, *2*(9), 249–259.

Martins, A. P., Gomes, H., Menezes, L., Costa, C., Pinto, H., Ribeiro, A. & Clain, T. C. (2021b). A História da Matemática no percurso formativo de futuros professores dos anos iniciais. In B. Silva, L. Ameida, A. Barca, M. Peralbo & R. Alves (Orgs.), *Atas do XVI Congresso Internacional Galego-Português de Psicopedagogia* (pp. 1399-1412). Universidade do Minho.

Martins, A. P., Gomes, H., Pinto, H., Ribeiro, A., Clain, T. C., Costa, C. & Menezes, L. (2021c). Teachers' Conceptions and Practices on the History of Mathematics in Teaching at 5th and 6th Grades in Portugal. In L. Chova, A. Martinez, I. Torres (Orgs.), *Proceedings of ICERI2021 Conference* (pp. 7078-7084). IATED.

Moyon, M. (2021). I would like to introduce history in my mathematics lessons but I do not know how to do it! *HPM Lecture*, 16 July. Retrieved from https://www.youtube.com/watch?v=NTIi44eCMLM

Siu, M. K. (2007). No, I don't use history of mathematics in my class. Why? In F. Furinghetti, S. Kaijser, & C. Tzanakis (Eds.), *Proceedings HPM2004 & ESU4* (pp. 268–277). Uppsala Universitet.