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Simple procedures of choice in multicriteria problems without precise information about the alternatives' values

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Abstract

The additive model of multiattribute value (or utility) theory is widely used in multicriteria choice problems. However, often it is not easy to obtain precise values for the scaling weights or the alternatives' value in each function. Several decision rules have been proposed to select an alternative under these circumstances, which require weaker information, such as ordinal information. We propose new decision rules and test them using Monte-Carlo simulation, considering that there exists ordinal information both on the scaling weights and on the alternatives' values. Results show the new rules constitute a good approximation. We provide guidelines about how to use these rules in a context of selecting a subset of the most promising alternatives, considering the contradictory objectives of keeping a low number of alternatives yet not excluding the best one.

Key words: Multi-Criteria Decision Analysis, MAUT/MAVT, imprecise/ incomplete/ partial information, ordinal information, simulation.

1 Introduction

Among the many approaches that were designed with the purpose of ranking a set of alternatives or with the purpose of selecting the best(s) one(s) taking into account multiple criteria, we address multiattribute utility theory - MAUT / multiattribute value theory - MAVT [25], which explores the idea of giving a value assessment to each alternative. This is a well-known and popular approach, although many other alternatives exist; for recent comprehensive reviews of other existing methods (outranking methods, analytic hierarchy process, etc.) the reader can see, e.g., [6, 17].

Usually it is assumed that the exact values for the parameters of multiattribute evaluation models are known or can be elicited from a decision maker. However, in many cases, this assumption is unrealistic or, at least, there may be advantages in working with less precise information. Several reasons are given in the literature that justify why a decision maker may wish to provide incomplete information [48, 13, 28, 14]: the decision may need to be taken under pressure of time and lack of data; some parameters of the model are intangible or non-monetary as they reflect social or environmental impacts, which can lead to the impossibility that the decision maker provides precise values with confidence; the decision maker has limited capacity to process information; the decision maker may find it difficult to compare criteria; the decision maker may not want to reveal his preferences in public or may not want to set his preferences, as they could change over the process; in each criterion, the performance of the alternatives can result from statistics or measurements (which generally are not completely accurate); some parameters reflect values or preferences of the decision maker, that he may consider difficult to express because he considers that it is difficult to assign precise numerical values to them; the performance of alternatives can depend on variables whose value is not known at the time of analysis; the uncertainty about what the future holds may also interfere with the system of values of decision maker; the information that would set the value of some parameters may be incomplete, not credible, contradictory or controversial. Some of these factors could be reduced with expense of time, discussions or money, but the decision maker may want to avoid incurring in these costs. Furthermore, experimental works [45, 10] show that different techniques for eliciting the values of parameters, given the same task and the same decision maker, may lead to different results. Working with models which require less effort from the decision makers is perhaps a way of fostering the adoption of formal methods for decision aiding. Thus, the decision maker may indicate only qualitative or ordinal information, or indicate intervals, instead of providing exact values for parameters. Motivated by these reasons, in this work we will consider decision problems in which decision aiding is provided based on such type of incomplete information.

The concern of working with incomplete information arises, naturally, in the context of the use of multiattribute value (or utility) functions. According to this technique [25, 49], it is necessary to begin by building a value function for each criterion, which expresses on a cardinal scale the value associated with each level of the scale where the criterion is measured. The value function may be increasing or decreasing as the level increases (for example, decreasing in the case of a cost), and often shows decreasing marginal values. In the case of the utility function it is also possible to model different attitudes towards risk. The most popular model for aggregating multiple value functions is the additive model, which advocates that, under some assumptions [25], the overall value of an alternative is the sum of value functions (one for each evaluation criterion), each of them weighted by a scale coefficient. These coefficients are the weights used on a weighted sum of objective functions. Henceforth we will refer to these coefficients simply as “weights”, though noting that they do not reflect directly the intuitive notion of importance of each criterion, because they are inseparable of the range for which the value function was defined. This is one of the most well-known methods among practitioners and researchers, it is simple to understand, and its theoretical properties are well studied (e.g., see [6, 24, 25, 49, 46]).

There are many methods that accept ordinal information (expressed as order relations), for example a ranking of the weights. Most of these methods focus on imprecision on weights, considering the precise value of each alternative in each criterion is known (e.g., [2, 13, 21, 42, 48, 44]). However, it is not always easy to elicit precise value of each alternative in each criterion. There are also methods that address imprecision on performance values (e.g., [22]), or are able to deal with imprecision on weights and on performance values simultaneously (e.g., [37, 41, 47]). During this work we will first address imprecision on performance values and later we will address imprecision on weights and on performance values simultaneously. Eliciting incomplete information about weights and about the value of each alternative in each criterion, although not precise, may be sufficient to increase the knowledge of the decision maker about the issue under analysis. The incomplete information can thus be used to provide some reference lines on the best choice or to provide some reference lines about how to select a subset of alternatives containing the most promising ones.

One of the research questions that arises in this context is to know how good are the proposed rules to select an alternative in the context of lack of precise information (for example, based on ordinal information), compared with an ideal situation in which the value of all parameters of the model is known. Usually this is studied using simulation (Monte-Carlo): generating randomly a large number of problems (criteria weights and value of each alternative in each criterion), determining the alternative with highest global value (based on the multiattribute model) and comparing this alternative with the alternative chosen by the rule under study, based on only part of the information. As examples of such comparisons we can cite, for example, [5, 42, 43, 44]. However, these works consider that only the weights are unknown, and it is important to extend this idea to the case where the value of the alternatives under each criterion are also unknown.

This paper presents new rules and simulation studies comparing different rules for choice when information about the weight of the criteria and about the value of each alternative in each criterion is incomplete. Unlike most previous research, we will not only focus on the best alternative according the used rule. More than using a rule to identify a single alternative, our aim is to test how the rules behave in a strategy of progressive reduction of the number of alternatives (as suggested by [13]), in the context of making a screening of the alternatives. Our aim is to test procedures to choose a subset of alternatives, based on incomplete information, for example a ranking of the value of the alternatives in each attribute, and then to observe how good the chosen alternatives are. The rules to be used try to conciliate the contradictory objectives of maintaining a minimum number of alternatives while ensuring that the chosen subset contains the best alternative. These experiments are designed to be comparable with previous studies. Hence, we test similar problem dimensions. Furthermore, we restrict ourselves to the case where the elicited information about these parameters is ordinal, i.e., a rank order.

In this work we will study the case in which the incomplete information does not refer only to weights, but also to the value of each alternative in each attribute, and it is possible to provide, in both cases, ordinal information instead of precise values for the parameters. For example, the decision maker can indicate that one alternative has higher value than another alternative in one criterion, without specifying how much. Larichev et al. [31] confirm the hypothesis that the decision is more stable if the information is elicited in an ordinal way.

In the next section we will present some of the existing approaches in the literature to deal with the use of ordinal information and other types of incomplete information. The rules tested are presented in detail in section 3, which also introduces the mathematical notation. In section 4 the conducted simulations are described, and results of such simulations are presented in Section 5. Section 6 shows an example of application of the decision rules. Section 7 presents some conclusions and some guidelines for future research.

2 Using Ordinal Information

In the literature it is possible to find different approaches which aim to skirt the difficulties associated with the elicitation of precise values for the parameters of models. We will present a brief overview of these approaches, particularly those involving ordinal information.

There are many methods that accept ordinal information, see for example [9]. The decision maker may indicate that a criterion is more important than some of the others, or that an alternative has better performance than another in a certain criterion, but not quantifying how much. This concern arises not only in methods based on the idea of a multiattribute value function, but also in methods based on different principles. For instance, determining exact values for weights is a difficulty associated also with outranking methods [40]. With the aim of reducing this difficulty, Bisdorff [8] proposes extending the principle of majority agreement (as implemented in outranking methods) to the context in which only ordinal information is available about the relative weight of the criteria. Within the category of outranking methods we could also mention QUALIFLEX [36], a method for ranking alternatives based on their rankings on several criteria and on the relative importance of these criteria, and ORESTE [39], which has been developed for situations where the alternatives are ranked according to each criterion and the criteria themselves are ranked according to their importance.

Other approaches not based on the idea of a multiattribute value function are Verbal Decision Analysis (VDA) [35], the TOMASO method [34], and distance-based approaches, to cite rather diverse examples. VDA uses ordinal information (e.g., more preferred, less preferred) and is more oriented for problems with a large number of alternatives and where the number of criteria is relatively small. ZAPROS and ORCLASS are two decision methods based on the principles of VDA: ZAPROS [29] aims at ranking a set of alternatives whereas ORCLASS [30] is used in classification problems. These methods make very few assumptions about the way the decision maker aggregates preferences. The TOMASO method can also be used for sorting or ranking based on evaluations of the alternatives on ordinal scales. It is based on Choquet integrals to allow interaction effects among criteria (e.g. synergy). Distance-based approaches attempt to find a ranking that is as close as possible (according to some distance) to a set of rankings (or partial rankings) provided as an input. As examples we can cite [12, 11, 19].

The indirect eliciting of preferences is used in the paradigm of ordinal regression, in which an underlying aggregation model is assumed, often a multi-attribute additive value function. According to this paradigm, initially information regarding holistic preferences within a set of reference alternatives is obtained and then the parameters for the model that maximize compatibility with this information are inferred. The inferred parameter values are then used to rank the alternatives. Figueira et al. [18] propose the GRIP method, which belongs to the class of methods based on the indirect elicitation of preferences and on the ordinal regression paradigm. GRIP can be seen as a generalization of the seminal UTA method [23] using additional information in the form of comparisons of intensity of preference between certain pairs of reference alternatives. The MACBETH methodology [4] is a multiattribute approximation requiring only qualitative judgments about value differences, with the aim of helping decision makers to quantify the relative attractiveness of alternatives.

Rather than inferring precise parameter values, Dias and Clímaco [13] presented the idea of using ordinal information to infer restrictions about these parameters values, assuming a multi-attribute additive value function was used. Their purpose was to use these constraints to obtain robust conclusions, i.e., to find the set of conclusions that is compatible with the provided information. The SMAA method [27] takes the reverse perspective by finding parameter values compatible with potential results: it emphasizes the exploration of weights space with the objective of discussing what type of parameter values makes an alternative preferred to another one. The SMAA-O [28] is a variant of SMAA for problems in which some or all criteria are measured in ordinal scales (it is known, in each criterion, which is the best alternative, the second best alternative and so on, but not the absolute measures).

To reconstitute the judgement of a decision maker concerning some alternatives provided as examples it is not necessary to infer numerical constraints or values. Greco et al. [20] presented a methodology to support multiattribute decision, the Decision Rule Approach, in which preferences are shaped in terms of decision rules "if ..., then ..." based on the dominance principle. The mathematical basis of the proposed methodology is the DRSA (Dominance-based Rough Set Approach). The authors state that decision makers accept more easily to provide information in terms of examples of decisions and look to simple rules to justify their decisions, than to supply precise values for parameters.

Also based on the concept of dominance, Iyer [22] explored the idea of extending dominance-based decision-making to problems with noisy evaluations. The author's idea was to eliminate alternatives which are dominated by any other alternative according to the multi-criteria evaluations, without assuming the aggregation method was known.

Much work has been developed for the case of MAVT/MAUT with incomplete information. Sage and White [41] proposed the model of imprecisely specified multiattribute utility theory (ISMAUT), in which precise preference information about both weights and utilities is not assumed. Malakooti [33] suggested a new algorithm for ranking and screening alternatives when there exists incomplete information about the preferences and the value of the alternatives. His proposed algorithm is very efficient, as many alternatives can be screened and ranked by solving a single mathematical programming problem. An extended version of Malakooti's work was presented by Ahn [1]. Park, Kim, and colleagues [16, 32, 37] provided linear programming characterizations of dominance and potential optimality for decision alternatives when information about values and/or weights is not complete, extended the approach to hierarchical structures [32], and developed the concepts of potential weak potential optimality (indicates if an alternative is sometimes better than the others given the incomplete information) and strong potential optimality (indicates if an alternative is always better than the others) [37]. White and Holloway [47] considered an alternative interactive selection process: a facilitator asks a decision maker questions and obtains responses that will be used to decide on the next question, aiming to eventually identify a most preferred alternative. In order to guide the facilitator in selecting what question to ask next and to determine when to terminate the question-response process, White and Holloway presented conditions that guarantee the existence of a question response policy that will identify a most preferred alternative in a finite number of questions.

In addition to all these methods, some authors (e.g., [15]) suggested the use of simple decision rules based on incomplete, but easy to elicit information. One of the possibilities described in the literature to deal with incomplete information on the weights is to select a weights vector w^* from a set of admissible weights W^* to represent that set and then to use w^* to evaluate the alternatives. Examples of this are the use of equal weights and the use of ROC (rank order centroid) weights, which are compared in the simulation study of Barron and Barret [5]. This study concludes that ROC weights provide a better approximation than the other weighting vectors. Another type of rules that have been proposed include optimization. It is possible to distinguish the following [42]: maximin rule (this rule consists in evaluating each alternative for its minimum guaranteed value, i.e., worst case), minimax regret rule (this rule consists in evaluating each alternative for the maximum loss of value with respect to a better alternative, i.e., a "maximum regret"), and central values rule (this rule consists in evaluating each alternative for the midpoint of the range of possible values). The idea of all these rules is to rank the alternatives, or to select an alternative, without requiring more information from the decision maker. Although none of these rules ensures that the alternative indicated as being the best one is the same that would result if precise values for weights were elicited, simulations show that in general the alternative selected is one of the best (e.g., [44]).

The work in this paper belongs to this last group of approaches of using rules based on information easy to elicit. Our objective is to rank the alternatives, or to select one alternative, without requiring precise information from decision maker. We will propose two new rules, based on the ideas of the ROC weights rule, to deal with incomplete information in the value of each alternative in each criterion. We will use Monte Carlo simulation to compare the results obtained when all the information is available with the results obtained when the proposed rules are used.

3 Notation and decision rules

3.1 Notation

The evaluation of a discrete set of m alternatives $A = \{a_1, ..., a_m\}$ is considered. The evaluation of each alternative is made according to each criterion, considering a set of n criteria (attributes) $X = \{x_1, ..., x_n\}$. Let $v_i(.)$ be the value function (or utility function - the difference here is not important) corresponding to attribute x_i. Consequently, $v_i(a_j) \in [0, 1]$ denotes the value of the alternative a_j according to the criterion x_i.

According to the additive aggregation model, the global (multi-attribute) value of an alternative $a_j \in A$ is given by:

$$v(a_j) = \sum_{i=1}^{n} w_i v_i(a_j) \tag{1}$$

where w_i represents the scale coefficient or "weight" associated with v_i. For these parameters we have:

$$w_1, ..., w_n \geq 0 \text{ and } \sum_{i=1}^{n} w_i = 1 \tag{2}$$

Without loss of generality we will consider that criteria weights are indexed by descending order, given ordinal information provided by a decision maker, for example, using "swings" [49, 15]. Thus, the set of all vectors of weights compatible with this information is:

$$W^* = \{(w_1, w_2, ...w_n) : w_1 \geq w_2 \geq ... \geq w_n \geq 0, \sum_{i=1}^{n} w_i = 1\} \tag{3}$$

We also consider that we have a ranking of the value of each alternative in each criterion: for each criterion the decision maker indicates which alternative has the highest value, which alternative has the second highest value, and so on. Let V^* be the set of the $n \times m$ matrices, having as elements the values $v_i(a_j)$ $(i = 1, ..., n;\ j = 1, ..., m)$, compatible with this information.

3.2 Decision rules

3.2.1 Introduction

Criteria weights are usually the parameters more difficult to elicit accurately [40]. Several authors have studied the case in which incomplete information refers only to criteria weights and it was verified that some decision rules based on ordinal information about the weights (for example, the decision maker indicates that a criterion weighs more than another) lead to good results [42, 43, 2, 44]. In [44], among other experiments, a set of Monte-Carlo simulations was carried out in order to see how different rules (ROC weights, maximin rule, minimax regret rule and central values rule) compared on a strategy of selecting the best alternative under each rule. The rules can be compared, for example, in accordance with the indicator "hit rate", which indicates the proportion of simulations in which the alternative chosen with a vector of supposedly true weights (i.e., the vector that would be obtained if precise values were elicited) coincides with the alternative indicated by the rule based on only ordinal information about the weights. The results indicate that the ROC weights are the best rule for this strategy (having a hit rate between 79% and 88%, for problem dimensions similar to those considered in this paper).

In this work we will also use ROC weights when the incomplete information refers to the weights. ROC weights are calculated using the following formula (assuming that the indices of criteria reflect their order, w_1 is the highest weight and w_n is the lowest one), defining the centroid of the simplex W^* (3):

$$w_i^{(ROC)} = \frac{1}{n} \sum_{j=i}^{n} \frac{1}{j},\ \ i = 1, ..., n. \tag{4}$$

3.2.2 Incomplete information on the value of each alternative in each attribute

If the decision maker finds it difficult to indicate the exact value of each alternative in each attribute, a natural idea would be to ask him for a ranking, e.g., "the alternative a_1 is the one which has the best value considering the attribute x_1, the alternative a_2 is the one which has the second best value considering the attribute x_1 and alternative a_3 is the one which has the third best value considering the attribute x_1".

Similarly to simulation experiments that studied the case in which only the weights were considered unknown, in this case it is also possible to compare the results obtained when we know the value of each alternative in each attribute with the results obtained when considering only a ranking. One possibility is to use ROC values for each attribute, i.e., the centroid of polytope defined by the ranking of the values on that attribute. This corresponds to equally spaced values; for attribute x_i, the ROC values are defined as follows $(i = 1, ..., n)$:

$$v_i^{(ROC)}(a_j) = \frac{m - r_i(a_j) + 1}{m + 1}, \; j = 1, ..., m. \tag{5}$$

where $r_i(a_j)$ represents the rank position of alternative a_j considering the attribute x_i and $r_i(a_j) < r_i(a_k) \Rightarrow v_i(a_j) \geq v_i(a_k)$.

Another idea is to ask the decision maker to provide, besides a ranking of the alternatives in each attribute, a ranking of the differences of value between consecutive alternatives. Imagine that a decision maker indicates that $v(x) > v(y) > v(z) > v(w)$, where x, y, z, and w are alternatives. Let $\Delta_{xy} = v(x) - v(y)$, $\Delta_{yz} = v(y) - v(z)$ and $\Delta_{zw} = v(z) - v(w)$. After that, the decision maker would need to provide a ranking of these Δ's. This approach might be based on a ranking of differences of value deduced from a rough drawing on a scale (for each individual attribute). Figure 1 shows an example of a hypothetical drawing done by a decision maker. It is not our objective, through this drawing, know the exact value of each alternative in each attribute, but only to get a sorting of the value of each alternative in each attribute, as well as a ranking of the differences of value in each attribute. This figure would allow to infer (checking with the decision maker if this was a correct reading) that $v_i(a_5) \geq v_i(a_4) \geq v_i(a_3) \geq v_i(a_2) \geq v_i(a_1)$ and $\Delta_4 \leq \Delta_3 \leq \Delta_2 \leq \Delta_1$, with $\Delta_1 = v_i(a_2) - v_i(a_1)$, $\Delta_2 = v_i(a_3) - v_i(a_2)$, $\Delta_3 = v_i(a_4) - v_i(a_3)$, $\Delta_4 = v_i(a_5) - v_i(a_4)$. We consider that $v_i(a_5) = 1$ and $v_i(a_1) = 0$. Note that this idea is similar to what is done in the decision support system DECAID [38]. However, contrary to what happens in DECAID, our objective is not to know the exact value of each alternative in each attribute, but only to obtain ordinal information about the position of alternatives and about the difference of value between them. Instead of asking for a rough drawing, the rank-order of the consecutive value differences might be asked directly. We will call ΔROC rule to the procedure of ranking the alternatives based on a formula similar to ROC weights. Note that, also for the $\Delta_1, ..., \Delta_{m-1}$ we have non-negativity restrictions and sum equal to one. Hence, for an attribute x_i, if the Δ_{ik} are indexed by decreasing order of magnitude, we have $(i = 1, ..., n)$:

$$\Delta_{ik}^{(\Delta ROC)} = \frac{1}{m-1} \sum_{j=k}^{m-1} \frac{1}{j}. \tag{6}$$

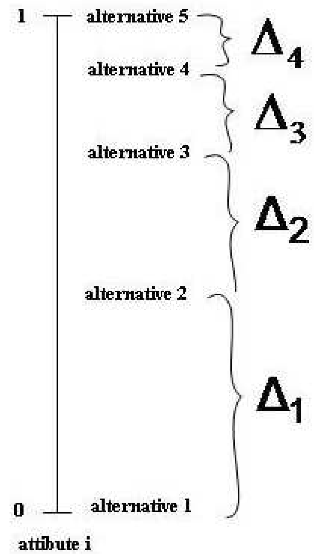

Figure 1: Example of a rough drawing on a scale for attribute i

After determining the $\Delta_1, ..., \Delta_{m-1}$, with the ΔROC rule, it is possible to calculate the value of each alternative in each attribute. For attribute x_i, the ΔROC values are defined as follows ($i = 1, ..., n$):

$$v_i^{(\Delta ROC)}(a_j) = \sum_{k=1}^{j-1} \Delta_{ik}^{(\Delta ROC)}, \; j = 1, ..., m. \tag{7}$$

4 Simulation

In the previous section we presented rules that can be used to select a promising subset of alternatives, given ordinal information about weights of the additive model, and ordinal information about the value of each alternative in each attribute. In this section we describe a sequence of experiments using Monte Carlo simulation to compare these rules. In these experiments we have considered situations with 5, 10, and 15 attributes, and 5, 10, and 15 alternatives. Similarly to [42] and [44], we have generated 5000 random problems for each problem dimension (after verifying that using a greater number of problems did not affect significantly the results).

The single-attribute values $v_i(a_j)$ were generated from a uniform distribution in the interval [0,1] and then normalized attribute-wise in such a way that the highest value in each attribute would be 1 and the lowest value would be 0. For each criterion, suppose v_i^{lo} and v_i^{hi} were the lowest and highest values among the m generated. Then, the normalized value of $v_i(a_j)$ is equal to $(v_i(a_j) - v_i^{lo})/(v_i^{hi} - v_i^{lo})$. The uniform distribution was also considered by [42] and [2].

The scaling weights were also generated according to an uniform distribution in W^* using the process described in [7]. To generate the weights for the n-attribute case, we draw $n-1$ independent random numbers from a uniform distribution on $(0,1)$ and rank these numbers. Suppose the ranked numbers are $r_{(n-1)} \geq ... \geq r_{(2)} \geq r_{(1)}$. The following differences can then be obtained: $w_n = 1 - r_{(n-1)}$, $w_{n-1} = r_{(n-1)} - r_{(n-2)}$, ..., $w_1 = r_{(1)} - 0$. Then, the set of numbers $(w_1, w_2, ..., w_n)$ will add up to 1 and will be uniformly distributed on the unit simplex defined by the rank-order constraints (3), after relabelling.

For each random problem, defined by a value matrix and a weights vector, the additive model provides the overall value of each alternative, which produces a ranking of the alternatives. This is what we call the supposedly true ranking, i.e., the ranking that would be obtained if cardinal information was used. On the other hand, each of the rules produces rankings using ordinal information about the weights vector and the values matrix. Comparing the ranking of the alternatives according to the supposedly true parameters with the ranking of the alternatives according to the used decision rule, we consider the following results:

- The position that the best alternative according to the true ranking reaches in the ranking generated by the used decision rule: this allows us to know the minimum number of alternatives that must be chosen, beginning by the top of the ranking provided by the rule, so that the true best alternative belongs to the chosen set.
- The position that the best alternative in the ranking generated by the rule reaches in the supposedly true ranking: this allows us to know how good the alternative chosen by the rule is in terms of the supposed true ranking.

Similarly to Barron and Barret [5] we also calculated the "value loss", i.e., the difference of multiattribute value between the alternative selected by the used decision rule and true best alternative, considering the supposedly true parameter values.

5 Results

5.1 Previous results

In [44] the authors considered the value of each alternative in each attribute was known, but the weights were unknown: only a ranking of the weights was available. A first set of experiments was carried out in order to see how the different rules compared on a strategy of selecting the best alternative under each rule. The experiments indicated the position reached by the alternative suggested by the rule in the supposedly true ranking. Detailed results related to the position that the best alternative under the different rules reaches in the supposedly true ranking were presented. The authors showed, for each rule and for each problem size, the average position in the supposedly true ranking and the proportion of cases where the reached position is 1, 2, 3, 4 or greater. Note that the proportion of cases where the reached position is equal to 1 corresponds to the hit rate. This set of results can also be useful to verify how the different rules compare on a strategy of selecting a subset of the best alternatives in accordance with each rule. For example, if the two best alternatives according to any of the rules are chosen, the proportion of cases where the supposed best alternative is one of the two chosen ones can be significantly higher than the hit rate.

In a strategy of progressive reduction of the number of alternatives, the objective is to retain the minimum number of alternatives for future analysis, without eliminating the best one. An interesting question is to know how many alternatives should be retained. To answer this question, in [44] the authors computed the position of the supposedly best alternative in the ranking produced by each rule. For each problem size, the average position of the supposedly best alternative in the ranking provided by each rule, and the proportion of cases in which the position was 1, 2, 3, 4, or greater were shown. The authors observed that the probability of retaining the supposed best alternative increases, as expected, with the number of alternatives that are retained. For example, using the ROC weights rule, to the considered dimensions, selecting two alternatives is sufficient to retain the supposed best alternative in 90% of the cases, and selecting three alternatives is not sufficient only in 5% of cases.

Next, we will present a similar study to obtain comparable results, first considering that the incomplete information refers only to the value of each alternative in each attribute, and afterwards considering that incomplete information refers to the weights and to the value of each alternative in each attribute simultaneously.

5.2 Incomplete information with respect to value of each alternative in each attribute

In this set of experiments we considered the weight of criteria was known, but we supposed that the decision maker indicated incomplete information about the value of each alternative in each criterion. We tested the ROC values rule (assuming that the decision maker ranked the alternatives) and the ΔROC values rule (assuming that the decision maker ranked the alternatives and ranked the difference between consecutive alternatives) for the value of each alternative in each criterion.

A first set of experiments was carried out in order to see how the different rules compare in a strategy of selecting the best alternative under each rule. These experiments indicate the position reached by the alternative suggested by the rule on the supposedly true ranking. Detailed results related to the position that the best alternative according to the ROC values and ΔROC values rules reached in the supposedly true ranking, are presented in tabular form in Table 1 (in this table TRUE ΔROC means the use of TRUE weights and ΔROC values and TRUE ROC means the use of TRUE weights and ROC values). This table shows, for each rule and for each size, the average position on the supposedly true ranking (the minimum position was always 1) and the proportion of cases where the position reached is 1, 2, 3, 4, or greater. The results indicate that the use of supposedly true weights and ΔROC values lead to a hit rate greater than 90%. With this rule, the hit rate increases with the number of alternatives. Using true weights and ROC values the hit rate varies between 76% and 81%. The use of ΔROC values leads to a significant increase in the hit rate when compared with the use of ROC values.

To answer the question "How many alternatives should be kept?", we need to know what is the position of the supposedly best alternative in the ranking produced by each rule. Table 2 shows, for each size, the average position of the supposedly best alternative in the ranking provided by each rule (minimum position was always 1) and the proportion of cases where the position is 1, 2, 3, 4, or greater. As we can see, the probability of retaining the supposedly best alternative increases with the number of alternatives that are retained. In all cases, selecting two alternatives will suffice in 93% of the cases while selecting three alternatives will not be sufficient in only 3% of cases. We can see that the additional information required from the decision maker by the ΔROC values rule is compensated by clearly superior results when compared with the ROC values rule.

In Table 3 it is possible to see the value loss of the different rules. In this table ROC ΔROC refers to the use of ROC weights and ΔROC values, ROC ROC refers to the use of ROC weights and ROC values, TRUE ΔROC refers to the use of TRUE weights and ΔROC values, and TRUE ROC refers to the use of TRUE weights and ROC values. Considering the weights (TRUE weights) are known and using ΔROC values, the average value loss varies between 0.0070 and 0.0316. The maximum value loss is a value between 0.0580 and 0.2455. Considering the weights are known and using ROC values, the average value loss varies between 0.0051 and 0.0139. The maximum value loss is a value between 0.0998 and 0.3123. Note that, in both cases, the average value loss is a small value.

		TRUE ΔROC						TRUE ROC					
n	m	average	% 1	% 2	% 3	% 4	% $\geq$ 5	average	% 1	% 2	% 3	% 4	% $\geq$ 5
5	5	1.10	91.00	8.20	0.72	0.08	0.00	1.25	78.78	17.44	3.36	0.40	0.02
5	10	1.09	91.82	7.36	0.68	0.14	0.00	1.28	78.66	16.38	3.64	1.04	0.28
5	15	1.08	93.32	5.88	0.62	0.18	0.00	1.29	79.02	15.30	4.14	1.06	0.40
10	5	1.10	90.76	8.46	0.74	0.04	0.00	1.24	80.94	14.92	3.38	0.70	0.06
10	10	1.09	91.58	7.54	0.76	0.12	0.00	1.32	77.38	16.02	4.50	1.58	0.52
10	15	1.08	93.00	6.28	0.64	0.08	0.00	1.31	77.76	16.04	4.50	1.16	0.54
15	5	1.10	90.80	8.36	0.80	0.04	0.00	1.26	79.20	16.64	3.38	0.70	0.08
15	10	1.09	91.88	7.12	0.92	0.06	0.02	1.31	76.84	16.7	4.64	1.26	0.56
15	15	1.08	92.80	6.40	0.64	0.14	0.00	1.30	79.00	14.74	4.40	1.44	0.42

Table 1: Position of the best alternative according to the ROC values and ΔROC values rule in the supposedly true ranking (n denotes the number of criteria and m the number of alternatives).

		TRUE ΔROC						TRUE ROC					
n	m	average	% 1	% 2	% 3	% 4	% $\geq$ 5	average	% 1	% 2	% 3	% 4	% $\geq$ 5
5	5	1.10	91.00	8.12	0.80	0.08	0.00	1.25	78.78	17.60	3.28	0.30	0.04
5	10	1.09	91.82	7.50	0.64	0.04	0.00	1.28	78.66	16.44	3.68	0.94	0.28
5	15	1.07	93.32	5.96	0.66	0.04	0.02	1.29	79.02	14.92	4.40	1.20	0.30
10	5	1.10	90.76	8.46	0.76	0.02	0.00	1.24	80.94	15.10	3.22	0.64	0.01
10	10	1.10	91.58	7.26	1.10	0.04	0.02	1.32	77.38	16.06	4.80	1.24	0.52
10	15	1.08	93.00	6.26	0.72	0.02	0.00	1.31	77.76	16.36	3.72	1.42	0.74
15	5	1.10	90.80	8.38	0.76	0.06	0.00	1.26	79.20	16.56	3.40	0.80	0.04
15	10	1.09	91.88	7.10	0.90	0.06	0.00	1.31	76.84	16.82	4.44	1.36	0.54
15	15	1.08	92.80	6.48	0.64	0.06	0.02	1.29	79.00	14.88	4.58	1.12	0.42

Table 2: Position of the supposedly best alternative in the ranking induced by the ROC values and ΔROC values rule.

		ROC ΔROC		ROC ROC		TRUE ΔROC		TRUE ROC	
n	m	average	maximum	average	maximum	average	maximum	average	maximum
5	5	0.0663	0.4926	0.0816	0.5351	0.0316	0.2455	0.0655	0.3123
5	10	0.0482	0.3177	0.0668	0.5143	0.0171	0.0994	0.0459	0.2377
5	15	0.0400	0.2364	0.0543	0.4223	0.0076	0.1755	0.0391	0.2105
10	5	0.0459	0.3605	0.0662	0.3808	0.0238	0.1195	0.0535	0.2602
10	10	0.0339	0.3466	0.0480	0.2845	0.0119	0.0705	0.0367	0.1996
10	15	0.0292	0.2780	0.0412	0.2804	0.0100	0.0607	0.0292	0.1599
15	5	0.0365	0.4018	0.0534	0.3167	0.0207	0.0991	0.0447	0.2341
15	10	0.0280	0.1751	0.0346	0.1946	0.0111	0.0693	0.0311	0.1922
15	15	0.0215	0.1869	0.0355	0.2080	0.0070	0.0580	0.0243	0.0998

Table 3: Value loss.

5.3 Incomplete information with respect to weights and with respect to value of each alternative in each attribute

In this section we considered both the weights of criteria and the value of each alternative in each criterion are unknown. The decision maker indicates only ordinal information about the weights and about the value of alternatives in each criterion, perhaps adding ordinal information about differences of value between consecutive alternatives in each criterion. We tested the rules that combine ROC weights / ROC values and combine ROC weights / ΔROC values. Table 4 shows the position of the best alternative under ROC weights / ROC values and ROC weights / ΔROC values rules in the supposedly true ranking (in this table ROC ΔROC means the use of ROC weights and ΔROC values and ROC ROC means the use of ROC weights and ROC values). Using ROC weights and ΔROC values the hit rate decreases with the number of alternatives. The results are obviously worse than in the case with known weights, because this is the situation in which less precise information is given. However, it should be noted that combining ROC weights/ ΔROC values gives results very close to the situation where it is assumed that the values of the alternatives are known (see [44]). Once again the additional information requested from the decision maker by ΔROC values rule is compensated by superior results when compared with the ROC values rule.

Table 5 shows the position of the supposedly true alternative in the ranking induced by the ROC weights / ROC values and ROC weights / ΔROC values rules. In the previous experiments that considered true weights and ΔROC values, results indicated the hit rate was greater than 90%. If we consider that we also do not know the weights, and used ROC weights, the results are also good (the hit rate is greater than 78%). Using the ROC weights and ΔROC values rule facilitates the elicitation of information and leads to a rapid identification of the most promising alternatives. Keeping 2 alternatives is enough to retain the supposed best one in at least 93% of the cases.

In Table 3 it is possible to see the value loss of the different rules. Considering the weights are unknown (using ROC weights) and using ΔROC values the average value loss varies between 0.0215 and 0.0663. The maximum value loss is a value between 0.1869 and 0.4926. Considering the weights are known and using ROC values the average value loss varies between 0.0355 and 0.0816. The maximum value loss is a value between 0.1946 and 0.5351. Considering both the weights and the values of each alternative in each attribute are unknown, the average value loss is also small.

		ROC ΔROC						ROC ROC					
n	m	average	% 1	% 2	% 3	% 4	% $\geq$ 5	average	% 1	% 2	% 3	% 4	% $\geq$ 5
5	5	1.21	83.12	13.40	3.06	0.42	0.00	1.32	74.98	19.22	4.74	0.98	0.08
5	10	1.29	79.44	14.84	3.88	1.42	0.42	1.41	72.92	17.80	6.00	2.42	0.90
5	15	1.31	78.98	14.56	4.18	1.34	0.60	1.47	71.26	17.86	6.52	2.70	0.92
10	5	1.21	83.02	13.90	2.66	0.38	0.04	1.31	75.98	18.14	4.62	1.18	0.08
10	10	1.25	81.42	13.82	3.42	1.02	0.32	1.42	72.26	18.24	60.6	2.26	1.18
10	15	1.29	80.50	13.90	3.48	1.20	0.92	1.46	71.84	17.92	6.08	2.36	1.80
15	5	1.19	84.40	12.44	2.76	0.40	0.00	1.32	76.20	17.52	4.92	1.12	0.24
15	10	1.23	83.24	12.72	2.74	0.94	0.36	1.39	74.02	17.40	5.48	2.12	0.98
15	15	1.26	81.40	13.30	3.76	1.02	0.52	1.39	75.22	15.80	5.74	1.86	1.38

Table 4: Position of the best alternative according to the ROC weights / ROC values and ROC weights / ΔROC values rules in the supposedly true ranking.

		ROC ΔROC						ROC ROC					
n	m	average	% 1	% 2	% 3	% 4	% $\geq$ 5	average	% 1	% 2	% 3	% 4	% $\geq$ 5
5	5	1.21	83.12	13.54	2.90	0.44	0.00	1.32	75.00	19.28	4.46	1.10	0.16
5	10	1.28	79.44	15.16	3.70	1.14	0.56	1.43	72.92	17.84	5.46	2.18	1.60
5	15	1.31	78.98	14.66	4.08	1.28	0.68	1.47	71.26	17.84	6.66	2.48	1.08
10	5	1.20	83.02	14.08	2.58	0.30	0.02	1.32	75.98	17.26	5.42	1.24	0.10
10	10	1.25	81.42	13.66	3.50	1.10	0.32	1.43	72.26	18.26	5.86	2.36	1.26
10	15	1.28	80.50	13.82	3.72	1.32	0.64	1.46	71.84	17.48	6.72	2.26	1.70
15	5	1.19	84.40	12.60	2.54	0.42	0.04	1.31	76.20	18.18	4.42	1.04	0.16
15	10	1.22	83.24	12.86	2.78	0.88	0.24	1.39	74.02	17.32	5.74	1.96	0.96
15	15	1.25	81.40	13.54	3.76	0.90	0.40	1.38	75.22	16.22	5.20	2.44	0.92

Table 5: Position of the supposedly best alternative in the ranking induced by the ROC weights/ ROC values and ROC weights / ΔROC values rules.

6 Example

Having compared some decision rules based on randomly generated problems, this section presents a comparison based on real data. With this intention, we used an example by Bana e Costa [3], based on a study of Keeney and Nair [26] about the choice of a location for a nuclear plant. In that study 9 alternatives were considered, which were evaluated through 6 attributes (cost, health and security, effect on the salmon population, socio-economic impact, aesthetics and biologic impact in the region).

The additive model was used with the following data for the value of each alternative in each attribute (matrix V) and for the weights associated with the value functions (vector w):

$$V = \begin{bmatrix} 0.957466 & 0.751 & 0.989533 & 0.7316 & 0.98 & 0.8135 \\ 1 & 0.8 & 0.989533 & 0.7145 & 0.98 & 0.8135 \\ 0.96799 & 0.875 & 0.989533 & 0.7149 & 0.86 & 0.802 \\ 0.959617 & 0.76 & 0.99782 & 0.5925 & 0.88 & 0.63 \\ 0.728686 & 0.78 & 0.993586 & 0.68535 & 0.76 & 0.469 \\ 0.60035 & 0.885 & 0.998013 & 0.56375 & 0.98 & 0.469 \\ 0.89758 & 0.74 & 0.99879 & 0.5725 & 1 & 0.7345 \\ 0.761342 & 0.945 & 0.9913 & 0.5275 & 1 & 0.7915 \\ 0.750121 & 0.91 & 0.9926 & 0.66915 & 1 & 0.9125 \end{bmatrix}$$

$$w = \begin{bmatrix} 0.347222 & 0.310764 & 0.189236 & 0.090278 & 0.051215 & 0.011285 \end{bmatrix}$$

Applying (1) to these data yields: $v(a_1) = 0.879$, $v(a_2) = 0.907$, $v(a_3) = 0.913$, $v(a_4) = 0.864$, $v(a_5) = 0.790$, $v(a_6) = 0.779$, $v(a_7) = 0.842$, $v(a_8) = 0.853$, and $v(a_9) = 0.853$. Hence, the alternative a_3 is the one which presents the highest global value.

If we consider that we did not know the exact values of each alternative under each attribute, we could use the ROC or ΔROC values rules. To use the ROC and ΔROC values rule it is necessary to normalize the values of each alternative under each attribute (matrix V'), in such a way that, in each attribute, the lowest value would be 0 and the highest value would be 1. Moreover, since we are changing the performance levels that correspond to levels of value 0 and 1, it is necessary to recalculate the weights (vector w') in order to compensate exactly the replacement of V by V'. These changes do not affect the ranking of alternatives in terms of the global value, continuing a_3 to be the best. The transformed data are:

$$V' = \begin{bmatrix} 0.893572 & 0.053659 & 0 & 1 & 0.916667 & 0.776776 \\ 1 & 0.292683 & 0 & 0.916218 & 0.916667 & 0.776776 \\ 0.919905 & 0.658537 & 0 & 0.918177 & 0.416667 & 0.750846 \\ 0.898954 & 0.097561 & 0.895214 & 0.318471 & 0.5 & 0.363021 \\ 0.321121 & 0.195122 & 0.437831 & 0.773395 & 0 & 0 \\ 0 & 0.707317 & 0.916064 & 0.177609 & 0.916667 & 0 \\ 0.743726 & 0 & 1 & 0.220480 & 1 & 0.598647 \\ 0.402832 & 1 & 0.190883 & 0 & 1 & 0.727170 \\ 0.374755 & 0.829268 & 0.331317 & 0.694023 & 1 & 1 \end{bmatrix}$$

$$w' = \begin{bmatrix} 0.578323 & 0.265502 & 0.007301 & 0.076790 & 0.051226 & 0.020858 \end{bmatrix}$$

Consider now that precise information about the weight of the criteria was not available at a certain stage of the analysis, because the decision maker felt more comfortable indicating only ordinal information. In such a case, we can use the ROC weights rule, which the simulations have shown to be the most promising rule. Extracting from vector w' only ordinal information about the weights, let us assume that we only knew that $w'_1 > w'_2 > w'_4 > w'_5 > w'_6 > w'_3$. Therefore, the weights vector to use the ROC rule would be the following:

$$w'^{(ROC)} = \begin{bmatrix} 0.408333 & 0.241667 & 0.027778 & 0.158333 & 0.102778 & 0.061111 \end{bmatrix}$$

With the ROC and ΔROC rules for the values we can determine an approximation for the values of the different alternatives in each attribute. The ROC and ΔROC values are the following (whenever there are ties this removes one variable; for instance in the 5th attribute we only have to consider 5 different levels):

$$V^{ROC} = \begin{bmatrix} 0.625 & 0.125 & 0 & 1 & 0.75 & 0.833333 \\ 1 & 0.5 & 0 & 0.75 & 0.75 & 0.833333 \\ 0.875 & 0.625 & 0 & 0.875 & 0.25 & 0.666667 \\ 0.75 & 0.25 & 0.666667 & 0.375 & 0.5 & 0.166667 \\ 0.125 & 0.375 & 0.5 & 0.625 & 0 & 0 \\ 0 & 0.75 & 0.833333 & 0.125 & 0.75 & 0 \\ 0.5 & 0 & 1 & 0.25 & 1 & 0.333333 \\ 0.375 & 1 & 0.166667 & 0 & 1 & 0.5 \\ 0.25 & 0.875 & 0.333333 & 0.5 & 1 & 1 \end{bmatrix}$$

$$V^{\Delta ROC} = \begin{bmatrix} 0.840327 & 0.054315 & 0 & 1 & 0.937500 & 0.841667 \\ 1 & 0.259821 & 0 & 0.905060 & 0.937500 & 0.841667 \\ 0.889435 & 0.599554 & 0 & 0.920685 & 0.270833 & 0.780556 \\ 0.855952 & 0.069940 & 0.911111 & 0.358780 & 0.416667 & 0.408333 \\ 0.214732 & 0.149256 & 0.502778 & 0.752827 & 0 & 0 \\ 0 & 0.633036 & 0.938889 & 0.214732 & 0.937500 & 0 \\ 0.688095 & 0 & 1 & 0.248214 & 1 & 0.650000 \\ 0.348363 & 1 & 0.241667 & 0 & 1 & 0.752778 \\ 0.294048 & 0.785268 & 0.4 & 0.698512 & 1 & 1 \end{bmatrix}$$

The results obtained using the different rules are presented in Tables 6 and 7 (in which the column TRUE TRUE refers to the use of true weights and true values, the column ROC TRUE refers to the use of ROC weights and true values, and so on, until the last column, which refers to the use of ROC weights and ΔROC values).

Alternative's value / Rule	TRUE TRUE	ROC TRUE	TRUE ROC	ROC ROC	TRUE ΔROC	ROC ΔROC
$v(a_1)$	**0.879**	0.678	0.527	0.572	0.643	0.662
$v(a_2)$	**0.907**	0.766	0.824	0.776	0.782	0.762
$v(a_3)$	**0.913**	0.769	0.766	0.713	0.774	0.729
$v(a_4)$	**0.864**	0.540	0.563	0.506	0.578	0.516
$v(a_5)$	**0.790**	0.313	0.223	0.255	0.225	0.257
$v(a_6)$	**0.779**	0.319	0.253	0.301	0.239	0.309
$v(a_7)$	**0.842**	0.506	0.374	0.395	0.489	0.491
$v(a_8)$	**0.853**	0.559	0.545	0.533	0.536	0.539
$v(a_9)$	**0.853**	0.636	0.490	0.566	0.507	0.595

Table 6: Value of different alternatives using different rules.

As expected, the results show that the rankings closer to the true one are those in which more precise information is used. It is preferable to have precise information about the values and ordinal information about the weights to have precise information about the weights and ordinal information about the values. If there is ordinal information about the values, then the ΔROC rule is superior to the ROC rule, as already had been verified in the conducted simulations. The alternative a_3, chosen in the original study, with the highest global value, is not the first for any of the rules (except for the TRUE ROC one). These coincide

Alternative / Rule	TRUE TRUE	ROC TRUE	TRUE ROC	ROC ROC	TRUE ΔROC	ROC ΔROC
a_3	**1st**	1st	2nd	2nd	2nd	2nd
a_2	**2nd**	2nd	1st	1st	1st	1st
a_1	**3rd**	3rd	5th	3rd	3rd	3rd
a_4	**4th**	6th	3rd	6th	4th	6th
a_8	**5th**	5th	4th	5th	5th	5th
a_9	**6th**	4th	6th	4th	6th	4th
a_7	**7th**	7th	7th	7th	7th	7th
a_5	**8th**	9th	9th	9th	9th	9th
a_6	**9th**	8th	8th	8th	8th	8th

Table 7: Position of different alternatives using different rules.

in pointing the second alternative of the real ranking, alternative a_2. However, this second alternative is also quite good, the difference of value between the two alternatives being very small. In this example, if the two best alternatives according to each rule were retained we would retain the two best ones.

7 Conclusions

This work presented a sequence of Monte-Carlo simulations with the aim of comparing different decision rules for the context where there exists only ordinal information about the weights of the attributes and about the values of each alternative in each attribute, considering the additive aggregation model for value functions. These experiments extend some experiments carried out by other authors, focusing on the calculation of hit rates and loss of value, since we tested strategies to select more than one alternative. These experiments also consider unknown the value of the alternatives in each attribute. The objective of this type of strategies is the simplification of the problem in terms of the number of alternatives, with the aim of studying it in more detail, or with the aim of eliciting more information.

The originality of this work, besides overtaking the focus on the best alternative for each rule, is the consideration of analogous decision rules for the case in which we have also ordinal information relatively the value of each alternative in each attribute. We proposed an adaption of the ROC rule for this purpose, and also a new rule that requires a little more information (but easily elicited, for example, by a rough drawing, as we suggested), the ΔROC values rule. If we consider that we do not know the value of each alternative under each attribute, our results show that ΔROC values rule lead to results clearly superior to the results of the ROC values rule.

As our experiments and example have shown, in the majority of the cases, using ordinal information leads to good results in the identification of the most promising alternatives. The best rule presented for cases without any cardinal information was ROC weights and ΔROC values. With this rule, the hit rate varies between 79% and 85%. The presented rule is good for selecting a subset of the most promising alternatives: selecting the two best alternatives according to this rule is sufficient in 93% of the cases or more, depending on the problem dimension, to retain the best alternative according to the true weights and values. The used elicitation of the information makes the cognitive task of the analysis easier, even if it is not very accurate. We recommend the use of incomplete information to identify a small subset containing the most promising alternatives, whenever it is predicted that it would be difficult to obtain precise values for all parameters. This may occur because the decision maker finds it cognitively difficult to express trade-offs cardinally, because time is scarce, because it is costly to assess or measure the performances of the alternatives, or for other reasons.

These conclusions should be read carefully, since the experiments were restricted to the case where the decision is based on a ranking of the criterion weights and on a ranking of the value of each alternative in each attribute. For the case where the set of acceptable weights and the set of acceptable values are defined by a set of general linear restrictions, it is possible that the ROC and ΔROC rules lose some power.

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