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Comparison of different rules to deal with incomplete information: perspectives of mediation

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Abstract

In bilateral Negotiation Analysis, the literature often considers the case with complete information. In this context, since the value (or utility) functions of both parties are known, it is not difficult to calculate the Pareto frontier (or efficient frontier) and the Pareto efficient solutions for the negotiation. Thus rational actors can reach agreement on this frontier. However, these approaches are not applied in practice when the parties do not have complete information. Considering that the additive value (or utility) function is used, often it is not easy to obtain precise values for the scaling weights or the levels' value in each issue. We compare four decision rules that require weaker information, namely ordinal information on weights and levels, to help a mediator suggesting an alternative under these circumstances. These rules are tested using Monte-Carlo simulation, considering that the mediator would be using one of three criteria: maximizing the sum of the values, maximizing the product of the excesses regarding the reservation levels, or maximizing the minimal proportion of potential. Simulations asses how good is the alternative chosen by each rule, computing the value loss with respect to the alternative that would be suggested if there was precise cardinal information and determining if the chosen alternative is efficient or, if not, how far is the nearest efficient alternative. We also provide guidelines about how to use these rules in a context of selecting a subset of the most promising alternatives, considering the contradictory objectives of keeping a low number of alternatives yet not excluding the best one. A further issue we investigate is whether using only ordinal information leads to treat one of the parties unfairly, when compared to a situation in which precise cardinal values were used instead.

Key words: Negotiation, Mediation, imprecise/ incomplete/ partial information, ordinal information, simulation.

1 Introduction

It is possible to distinguish four types of procedures for deciding when several decision makers are involved [7]: individual decision-making in a group setting, hierarchial decision making, group decision-making and negotiation. In individual decision-making in a group setting the decision maker utilizes knowledge of experts, advisers or stakeholders during the process. Only one person is responsible for the decision made, but all group members participate in the process. In hierarchial decision making it is possible to distinguish two cases: centralized and decentralized. In the centralized one, there is one set of objectives representing the top-level decision maker, who has full control over the lower-level members. In the decentralized case, each member independently controls subsets of the decision variables and objectives and is responsible for his decision which serves as input to the higher-level one. In group decision-making each group member participates in the process and is partly responsible for the final decision. There usually is an overall goal which is accepted by all the members, but they differ in the ways of how this goal should be achieved. In negotiation, one negotiator represents one party and is responsible for the decision before this party and not before the other one(s). There is a conflict of interests because parties have separate and conflicting objectives and they have different needs which they want to satisfy. Negotiation is the chosen way to resolve a conflict out of necessity and not out of effectiveness or efficiency. Our focus in this paper is on situations involving two individual negotiators interacting with the assistance of a mediator. We will consider the externally prescriptive perspective [12], according to which the objective is to determine how mediators or arbitrators can act to help parties to negotiate in a balanced and impartial way. The general goal of this work is to contribute with new methodologies to support a mediator, enabling him or her to advise negotiators with "good" alternatives.

Usually it is assumed that the value of parameters of the different models is known or can be asked to a negotiator. However, in many cases, this assumption is unrealistic or, at least, there might be advantage in working with less precise information. We will consider that negotiators feel more "comfortable" providing ordinal information. Thus, a negotiator could be allowed to indicate only qualitative or ordinal information, or indicate intervals, instead of indicating precise values for the parameters of the used model. It is possible to present different reasons why negotiators may wish to provide incomplete/imprecise/partial information (see for example [20] and [10]). There are some approximations presented in the literature to work with incomplete information in negotiation processes. Vetschera [19] proposed a method to measure the amount of information that is available during the negotiation. Vetschera considered integrative negotiation and considered that the information was obtained in an implicit way through the offers. The method is based on the domain criterion and can be used whether a compromise is reached or not. Clímaco and Dias [4] proposed an extension of the methodology of the software VIP-G for negotiation processes. The methodology presented adjusts itself to problems of negotiation between two parties in the discrete case. The analysis is based on the weights space and uses the concept of convergence paths. This methodology is suited to situations where the parties reach agreement on what issues are to maximize and minimize. Lai et al. [11] presented a model that considers Pareto efficiency (efficient solutions are solutions where it is not possible to improve the value for one party without worsening the value to one of the other parties) and computational efficiency, to situations where information is incomplete, the value functions are not linear and are not explicitly known. Ehtamo et al. [5] presented a class of interactive methods, called constraint proposal methods, to find Pareto efficient solutions through common tangent hyperplanes, which fit negotiations of two parties with two or more continuous subjects. Heikanen [6] proposed a method to determine Pareto efficient solutions in negotiations with multiple parties about continuous subjects. In this method it is not required that negotiators know the value functions of other parties or that someone outside the negotiation knows all the value functions.

We will consider a setting of bilateral integrative negotiation over multiple issues in the discrete case. The main assumption we will make is that the preferences of both parties can be roughly modelled by an additive value (or utility) function, as assumed in Raiffa's Negotiation Analysis [12]. The construction of the proposals consists in the identification of issues to solve, in the specification of the possible levels of resolution for each subject, and in the specification of the scores of each possible combination of levels (scores which can be obtained through the additive value / utility model).

Instead of assuming that negotiators are able to define their value functions precisely, we assume we only elicit ordinal information about the preferences of each party: about the weights of the different issues and about the value of the different levels in each issue. Our objective is that, with this ordinal information, a mediator could suggest one or more alternatives based on some decision rules we will present in the next section. To compare these rules we will use three criteria: maximizing the sum of the values, maximizing the product of the excesses regarding the reservation levels and maximizing the minimal proportion of potencial. We will use simulation to test if the suggested alternative(s) is (are) "good" alternative(s). Our objective is not to compare criteria but to compare rules. The choice of a criterion depends on the preferences of the mediator, as all of them present advantages and disadvantages. For example, maximizing the sum of the values may not be attractive to many because of the inequality that usually results.

The main contribution of this paper is to show that it is possible to obtain good results considering incomplete information regarding the preferences of the parties. The tested rules and the used criteria are presented in detail in the next section, which also introduces the mathematical notation. In section 3 the conducted simulations are described, and results of such simulations are presented in Section 4. We will finish in section 5 with some conclusions and with some lines for future research.

2 Notation, criteria and decision rules

2.1 Notation

This work considers integrative negotiation among two parties over n issues. We further assume that an additive form of the value function is appropriated (see [8] and [21] for necessary and sufficient conditions for such additivity). Let x be one proposal. The global value of x, for a negotiator k ($k = 1, 2$), can be obtained by:

$$V^k(x) = \sum_{j=1}^{n} V_j^k(x) = w_1^k v_1^k(x) + w_2^k v_2^k(x) + ... + w_n^k v_n^k(x) \tag{1}$$

where $V_j^k(x) = w_j^k v_j^k(x)$, $v_j^k(x)$ represents the value of the proposal x concerning the j^{th} issue and w_j^k represents the scale coefficient or "weight" of the value function $v_j^k(.)$, for party k. We assume that:

$$0 \leq w_j^k \leq 1, j = 1, ...n \text{ and } \sum_{j=1}^{n} w_j^k = 1. \tag{2}$$

Without loss of generality, we consider that the indices of the issues are coded such that the weights are in decreasing order. Thus, the set of all vectors of weights compatible with this information, for party k ($k = 1, 2$) is:

$$W^k = \{(w_1^k, w_2^k, ...w_n^k) : w_1^k \geq w_2^k \geq ... \geq w_n^k \geq 0, \sum_{i=1}^{n} w_i^k = 1\} \tag{3}$$

We also consider that, for each negotiator, we have a ranking of the value of each level in each issue, i.e., negotiators indicate if each issue is a maximizing one or a minimizing one. Let V^k be the set of matrices, having as elements the values $v_i^k(l_{ji})$, where l_{ji} is the level j in issue i ($i = 1, ..., n$ and $j = 1, ..., m_i$, where m_i is the number of levels of issue i), for party k ($k = 1, 2$), compatible with this information.

2.2 Criteria

We can assume that for each party some contracts are acceptable whereas others are not. In the currency of the scoring system, each party is assumed to have a reservation value associated with its BATNA (best alternative to a negotiated agreement). The reservation value will specify the minimum value that will be acceptable for each party [12]. We will assess which would the most promising proposals be according to some well-known arbitration criteria (see [12]):

- Maximizing the sum of the values: according to this criterion the chosen alternative is the one for which the sum of the values is maximum (i.e., the alternative x for which $V^1(x) + V^2(x)$ is maximum).
- Maximizing the product of the excesses regarding the reservation levels: according to this criterion the chosen alternative is the one for which the product of the excesses regarding the reservation levels is maximum (i.e., the alternative x for which $(V^1(x) - RV^1) * (V^2(x) - RV^2)$ is maximum, where RV^k is the reservation level of party k, $k = 1, 2$).
- Maximizing the minimal proportion of potential (PoP): according to this criterion the chosen alternative is the one for which the minimal PoP is maximum. For each party, the PoP of an alternative is obtained dividing the difference between the value of the alternative and the reservation value by the difference between the maximum value admissible for the party and the reservation value (i.e., for alternative x and for party k, $PoP^k(x) = \frac{V^k(x) - RV^k}{V^{maximum^k} - RV^k}$, where $V^{maximum^k}$ is the maximum value for party k in the range of admissible agreements).

If the reservation levels are equal to zero, the criterion maximizing the product of the excesses regarding the reservation levels corresponds to the Nash criterion, in which the objective is to maximize the product of the values of both parties (i.e., maximize $V^1(x) * V^2(x)$).

2.3 Decision rules

Criteria weights are usually the parameters more difficult to accurately elicit [13]. Several authors have studied the case in which incomplete information refers only to criteria weights, in cases of multi-attribute choice or ranking. It was verified that some decision rules based on ordinal information about the weights (for example, the decision maker indicates that a criterion weighs more than another) lead to good results [1, 14, 15, 16]. One of the possibilities described in the literature to deal with incomplete information on the weights is to select a weights vector, w, from a set of admissible weights W to represent that set and then to use w to evaluate the alternatives. The study of Barron and Barret [2] concludes that ROC weights provide a better approximation than other weighting vectors. In this work we will extend this idea to negotiation processes, using ROC weights when the incomplete information refers to the issues weights. ROC weights are calculated using the following formula (assuming that the indices of issues reflect the ranking of the weights, with w_1^k being the highest weight and w_n^k being the lowest one), defining the centroid of the simplex W^k (3), for party k ($k = 1, 2$):

$$w_i^{k(ROC)} = \frac{1}{n}\sum_{j=i}^{n}\frac{1}{j},\ i = 1, ..., n. \tag{4}$$

Considering the information regarding the value of the different levels related to the different issues we will consider three cases: negotiators can indicate the exact value of each level in each issue (values known), negotiators can indicate a ranking of the levels in each issue and a ranking of the differences of value between consecutive levels in each issue and negotiators can indicate only a ranking of the levels in each issue.

Considering that negotiators can indicate a ranking of the levels in each issue and a ranking of the differences of value between consecutive levels in each issue we will use the ΔROC values rule (see [17]). Hence, for an issue i, if the Δ_{ip}^k are indexed by decreasing order of magnitude, we have ($i = 1, ..., n$):

$$\Delta_{ip}^{k(\Delta ROC)} = \frac{1}{m_i - 1}\sum_{j=p}^{m_i-1}\frac{1}{j}. \tag{5}$$

where m_i is the number of levels.

After determining $\Delta_{i1}^{k(\Delta ROC)}, ..., \Delta_{i(m_i-1)}^{k(\Delta ROC)}$ with the ΔROC rule, it is possible to calculate an approximate value of each level in each issue. For issue i, and for level l_{ji}, the ΔROC values are defined as follows ($i = 1, ..., n$):

$$v_i^{k(\Delta ROC)}(l_{ji}) = \sum_{p=1}^{j-1}\Delta_{ip}^{k(\Delta ROC)},\ j = 1, ..., m_i. \tag{6}$$

If the negotiators can indicate only a ranking of the levels in each issue one possibility is to use ROC values for each issue (see [17]). This corresponds to the centroid of polytope defined by the ranking of the level values on that issue. This corresponds to equally spaced values; for issue i, and for party k ($k = 1, 2$) the ROC values are defined as follows ($i = 1, ..., n$):

$$v_i^{k(ROC)}(l_{ji}) = \frac{m_i - r_i^k(l_{ji}) + 1}{m_i + 1},\ j = 1, ..., m. \tag{7}$$

where $r_i^k(l_{ji})$ represents the rank position of the level l_{ji} considering the issue i and $r_i^k(l_{ji}) < r_i^k(l_{pi}) \Rightarrow v_i^k(l_{ji}) \geq v_i^k(l_{pi})$, for party $k = 1, 2$.

Another possibility, if negotiators can indicate only a ranking of the levels in each issue, is to make an approximation based on a linear value function [18]. In such cases we will consider that value functions are linear although this is a rather strong assumption. However some authors consider that the simpler linear function is preferable for several reasons (see [19]). Some of the presented reasons are: more general function also requires more parameters, as only a limited number of observations is available, the number of parameters should be kept as small as possible (this is an advantage of the linear form, which requires no additional parameters for the marginal value functions); any nonlinear function also entails the risk of mis-specification, thus it could happen that even a function requiring more parameters than the simple linear form would not provide a better approximation to the negotiator's true preferences.

3 Simulations

To test the presented rules we used two cases as templates for generating random examples: case Nelson vs Amstore presented in [12] and case Itex vs Cypress used in the InterNeg project (written by David Cray of Carleton University) [9].

In Nelson vs Amstore case, there are two parties in negotiation: Amstore and Nelson. Nelson has a construction firm and he is negotiating with a retail chain (Amstore) to build a new store. There are three issues: price (10, 10.5, 11, 11.5 or 12 thousand dollars), design (basic or improved) and time (20, 21, 22, 23, 24, 25 or 26 days). There are a total of 70 possible alternatives. For Nelson, price and time are maximizing issues and design is a minimizing one, while for Amstore it is the opposite. Therefore, the preferred alternative for Amstore is a price of 10 thousand dollars, an improved design and a period of 20 days. The preferred one for Nelson is a price of 12 thousand dollars, a basic design and a period of 26 days. The reservation value (on a 0-100 value scale) for Nelson is equal to 60 and for Amstore it is equal to 20. Note that in this example, using ROC values coincides with the case in which an approximation of the values is done considering linear value functions. This happens because the different levels of the issues are equally spaced, and the ROC values rule coincide with the use of equal spaced values.

In Itex vs Cypress case, there are two companies: Itex Manufacturing, a producer of bicycle parts, and Cypress Cycles that builds bicycles. Both sides negotiate over the same four issues: the price of the bicycle components (3.47$, 3.71$, 3.98$, 4.12$ or 4.37$), delivery schedules (20 days, 30 days, 45 days or 60 days), payment arrangements (upon delivery, 30 days after delivery or 60 days after delivery), and terms for the return of defective parts (full price, 75% refund with 5% spoilage or 75% refund with 10% spoilage). For each issue there is a pre-specified set of options, i.e., issue levels. Altogether, there are 180 complete and different potential offers (alternatives) that specify levels for all four issues. For Itex, price, delivery and terms of return are maximizing issues and payment is a minimizing one, while for Cypress it is the opposite. Therefore, the preferred alternative for Itex is a price of 4.37$, delivery in 60 days, upon delivered payment and 75% refund with 10% spoilage. The preferred one for Crypess is a price of 3.37$, delivery in 20 days, payment 60 days after delivery and full price refund. There is not any information regarding the reservation level of the parties. Note that in the fourth issue, and when we are making an approximation of level's value using linear value functions, it is not easy to know what is the value that should be assign to the second level. In the absence of a better reason we chose to use the value 0.5.

To use the two last criteria presented in section 2.2 (maximizing the product of the excesses regarding the reservation levels and maximizing the minimal PoP) it is necessary to know the reservation level for both parties in negotiation. In both examples we considered initially that the reservation levels are equal to zero (this corresponds to the case in which there is no reservation level). In a second stage we suppose that we know the reservation value for Nelson is equal to the value of the alternative 25 - (10.5, basic, 23). In the original problem this is one of four alternatives that for Nelson presents a value of 60 and is the most equilibrated (presents less extreme positions). Suppose also that the reservation value for Amstore is equal to the value of the alternative 64 - (12, basic, 20). In the original problem this alternative presents for Amstore a value of 20. Since in the Itex vs Cypress case there is no information regarding the reservation levels we will consider first that the reservation levels are equal to the values of the alternative 52 - (3.71$, 30 days, 60 days after delivery, full price) and the value of the alternative 129 - (4.12$, 45 days, upon delivery, 75% refund with 10% spoilage), for Itex and Cypress, respectively, and after we will consider that the reservation levels are equal to the value of the alternative 86 - (3.98$, 30 days, 30 days after delivery, 75% refund with 5% spoilage) and the value of the alternative 95 (3.98%, 45 days, 30 days after delivery, 75% refund with 5% spoilage), for Itex and Cypress, respectively. These alternatives correspond, approximately, to the 25th percentile and to the 50th percentile for Cypress and Itex.

To generate random examples for these templates, the level values $v_i^k(l_{ji})$ were generated from a uniform distribution in the interval [0,1] and then normalized in such a way that the highest value in each issue would be 1 and the lowest value would be 0. For each issue, suppose that v_i^{klo} and v_i^{khi} were the lowest and highest values among the m_i generated for party k. Then, the normalized value of $v_i^k(l_{ji})$ is equal to $(v_i^k(l_{ji}) - v_i^{klo})/(v_i^{khi} - v_i^{klo})$, for $k = 1, 2$. The scaling weights were also generated according to an uniform distribution in W^k using the process described in [3]. To generate the weights for the n-issue case, we draw $n-1$ independent random numbers from a uniform distribution on $(0, 1)$ and rank these numbers. Suppose the ranked numbers are $r^k_{(n-1)} \geq ... \geq r^k_{(2)} \geq r^k_{(1)}$ for party k. The following differences can then be obtained: $w_n^k = 1 - r^k_{(n-1)}$, $w^k_{n-1} = r^k_{(n-1)} - r^k_{(n-2)}$, ..., $w_1^k = r^k_{(1)} - 0$. Then, the set of numbers $(w_1^k, w_2^k, ..., w_n^k)$ will add up to 1 and will be uniformly distributed on the unit simplex defined by the rank-order constraints (3), $k = 1, 2$.

For each random problem, defined by a level value matrix and a weights vector, the additive model provides the overall value of each alternative, which produces a ranking of the alternatives. This is what we call the supposedly true ranking, i.e., the ranking that would be obtained if this cardinal information was known. On the other hand, each of the rules produces rankings using ordinal information about the weights vector and the level values matrix. We will consider that:

- $x_{real(sum)}$: is the real best alternative according to the criterion maximizing the sum of the values;
- $x_{real(product)}$: is the real best alternative according to the criterion maximizing the product of the excesses regarding the reservation levels;
- $x_{real(PoP)}$: is the real best alternative according to the criterion maximizing the minimal PoP;
- $x_{rule(sum)}$: is the best alternative provided by the rule according to the criterion maximizing the sum of the values;
- $x_{rule(product)}$: is the best alternative provided by the rule according to the criterion maximizing the product of the excesses regarding the reservation levels;
- $x_{rule(PoP)}$: is the best alternative provided by the rule according to the criterion maximizing the minimal PoP.

We started by determining the value loss, i.e., the difference between the real value of the best alternative and the real value of the alternative chosen by the rule (considering the criteria maximizing the sum of the values, maximizing the product of the excesses regarding the reservation levels and maximizing the minimal PoP), in cases in which the two alternatives did not match. For example, in the case of the criterion maximizing the sum of the values, the value loss is given by $(V^1(x_{real(sum)}) + V^2(x_{real(sum)})) - (V^1(x_{rule(sum)}) + V^2(x_{rule(sum)}))$. For the other criteria the idea is the same. This allows us to know if the alternatives chosen by the rules have global value much inferior to the best alternatives in reality. Note that the maximum possible value loss is equal to 200 to the criterion maximizing the sum of the values, is equal to 10000 to the criterion maximizing the product of the excesses regarding the reservation levels and is equal to 1 to the criterion maximizing the minimal PoP.

To see if the alternative chosen by the rule is a good alternative, we determined also the proportion of cases in which the chosen alternative is efficient. Remember that efficient solutions are solutions where it is not possible to improve the value for one party without worsening the value to one of the other parties. When the chosen alternative is not efficient we determined the distance between the chosen alternative and the nearest efficient alternative. We determined the distance between the two alternatives using two type of distances: L_2 and L_∞. Let $x = (x_1, x_2)$ and $y = (y_1, y_2)$ be two proposals. The L_p $(p = 2 \ and \ \infty)$ distances can be calculated by:

- L_2 (Euclidean distance): $d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$
- L_∞ (Tchebychev distance): $d(x, y) = \max\{|x_1 - y_1|, |x_2 - y_2|\}$

We determined also the difference between the real value of the best alternative and the real value of the alternative chosen by the used criterion, for party 1 and party 2. For example, for the criterion maximizing the sum of the values we determined $(V^1(x_{real(sum)}) - V^1(x_{rule(sum)}))$ and $(V^2(x_{real(sum)}) - V^2(x_{rule(sum)}))$. This enables us to know whether using only ordinal information leads to treat one of the parties unfairly, when compared to a situation in which precise cardinal values were used instead. We consider that the alternative obtained by a rule, x, is a fair (unbiased) approximation if $((V^1(x_{real(sum)}) - V^1(x_{rule(sum)}) \approx (V^2(x_{real(sum)}) - V^2(x_{rule(sum)}))$

Comparing the ranking of the alternatives according to the supposedly true parameters with the ranking of the alternatives according to the used decision rule, we consider the following results:

- the position that the best alternative according to the true ranking reaches in the ranking generated by the used decision rule (this allows us to know the minimum number of alternatives that must be chosen, beginning by the top of the ranking provided by the rule, so that the true best alternative belongs to the chosen set);
- the position that the best alternative in the ranking generated by the rule reaches in the supposedly true ranking (this allows us to know how good the alternative chosen by the rule is in terms of the supposed true ranking).

The position that the best alternative according to the true ranking reaches in the ranking generated by the used decision rule allows us to assess the strategy of retaining $k < m$ alternatives instead of only one. The position that the best alternative in the ranking generated by the rule reaches in the supposedly true ranking complements the other results that show if the alternative chosen by the rule is a good alternative (the value loss, the proportion of cases in which the chosen alternative is efficient and the distance between the chosen alternative and the nearest efficient alternative).

In the next section we will present the results of 5000 iterations per simulation (after verifying that using a greater number of problems did not affect significantly the results).

4 Results

In this section we will present the results of the simulations described in the previous section. During this section and in the tables referred, ROC TRUE refers to the use of ROC weights and TRUE values, ROC ΔROC refers to the use of ROC weights and ΔROC values, ROC ROC refers to the use of ROC weights and ROC values and ROC Linear refers to the use of ROC weights and Linear values. All the referred tables are presented in Appendix. During this section we will present results of statistics tests considering a significance level equal to 0.01.

4.1 Examples based on the Nelson vs Amstore template

4.1.1 No information available about the reservation levels of the parties

In Table 1 it is possible to see the value loss of the different rules considering the different criteria, i.e., the difference between the real value of the real best alternative and the real value of the alternative chosen by the rule. This table shows the average, the standard deviation, the maximum, the 75th percentile and the 95th percentile of the value loss. As expected, the worst results are obtained using ROC ROC / ROC Linear (because these are the cases in which less information is required from negotiators). Using ROC TRUE or ROC ΔROC the results are very similar. The value loss is not very high even considering ROC ROC / ROC Linear.

Table 2 presents the proportion of cases in which the chosen alternative is efficient (using the different rules and considering the different criteria). Using the ROC TRUE the proportion is highest and using the ROC ROC / ROC Linear the proportion is lowest (as expected). In the cases in which the chosen alternative is not efficient it is interesting to know the distance to the nearest efficient alternative. The results are presented in Tables 3 and 4, using the L_2 and L_∞ distances, respectively. In these tables it is possible to see the average, the standard deviation, the maximum, the 75th percentile and the 95th percentile of the distance. Considering the Euclidean distance, the best results (lower distances) are obtained using the ROC TRUE and ROC ΔROC (these rules presented very similar results). Using the other distance (L_∞), the conclusions are the same. Note that the different rules yield worse results for the criterion of maximizing the minimal PoP than the other criteria. The proportion of cases in which the chosen alternative is efficient is not very high and, besides that, when the chosen alternative is not efficient the distance to the closest efficient alternative is not so small.

We computed some statistics tests to check if the difference between the ROC TRUE and ROC ΔROC rules is significative (these are the two rules that provide the best results). We determined the p - value, i.e., the lowest level of significance at which the hypothesis of equality of the rules can be rejected. The results are presented in Table 5. In this table it is possible to see the p - value when comparing the average of the value loss, the average of the Euclidean distance, the average of the L_∞ distance of the two rules and also the proportion of cases in which the chosen alternative is efficient. As it is possible to see considering the proportion of cases in which the chosen alternative is efficient, and using the criterion maximizing the sum of the values, the ROC TRUE rule is better. In all the other cases the difference between the two rules is not significative. It seems that the ROC ROC rule is the one which provide worst results. We also computed some statistics tests to check if the difference between the ROC ROC and ROC ΔROC rules are significative. As the p-values are equal to zero or very close, we can conclude that the ROC ΔROC rule is better than the ROC ROC rule.

We can see in Tables 6 and 7 the difference between the real value of the best alternative and the real value of the alternative chosen by the used criteria, for Nelson and Amstore, respectively. In these tables it is possible to see the average of the difference. As it is possible to see, the best approximation for Nelson would occur if maximizing the minimal PoP was used, because the average difference is small. Even using this criterion, in average, Nelson loses with respect to the real best alternative, because the difference is positive. When maximizing the sum of the values, worse approximations for Nelson appear. The best results (smaller differences) are obtained using the ROC TRUE rule and the worst (bigger differences) are obtained using ROC ROC / ROC Linear rule. Using the the sum of the values criterion, in average, Amstore wins comparing with the real best alternative. For Amstore, the least favorable approximations are obtained using the criterion maximizing the minimal PoP. It is possible to see that, in average, both parties lose in using the rules instead of real values, except Amstore with the criterion maximizing the sum of the values. It is possible to consider that the criterion maximizing the minimal PoP provides a fair (unbiased) approximation, because the considered difference is similar for both parties. The same does not happen for the criterion maximizing the sum of the values.

Detailed results related to the position that the best alternative according to the different rules reached in the supposedly true ranking, are presented in Table 8. This table shows, for each rule and for each criteria, the average position on the supposedly true ranking (the minimum position was always 1) and the proportion of cases where the position reached is 1, ≤ 2, ≤ 3, ≤ 4, ≤ 5, ≤ 10 and ≤ 20. This table gives us the information to know how many alternatives should be chosen to guarantee the retention of the supposed best alternative. As expected, as the total number of alternatives is equal to 70, retaining only one alternative is not sufficient in the majority of the cases. However, even then, we can consider that the "hit rate", i.e., the proportion of cases in which the best alternatives in the two rankings coincide, is not very bad. Retaining 20 alternatives (close to 30% of the total number of alternatives) the probability of retaining the best one is almost always higher than 90%. The hit rate using the ROC ROC / ROC Linear values and the criterion maximizing the sum of the values is surprisingly high. This happens because the high number of ties (using ROC / Linear values, the value functions of both parties are symmetrical, and because of that, when the weights match there are a lot of alternatives with equal sum of values).

Results relatively to the position of the best alternative using the different rules in the supposedly true ranking are shown in Table 9. These results enable us to know how good is the alternative chosen by each rule in terms of the supposedly true ranking. Note that the value of the hit rate should be the same considering the position of the supposedly best alternative in the rankings induced by the rules and the position of the best alternative using the different rules in the supposedly true ranking. However this did not happen because, if there are ties in the first place of the ranking, which is frequent when the different rules are used, the chosen alternative is the first one with position in the ranking equal to one. Because of that, the hit rate considering the position of the supposedly best alternative in the ranking induced by the different rules is superior or equal than the hit rate considering the position of the best alternative using the different rules in the supposedly true ranking. These results complement the ones regarding the value loss, the proportion of cases in which the chosen alternative is efficient and, if the alternative is not efficient, the distance to the nearest efficient alternative. In more than 81% of the cases the alternative chosen by the rule is one of the best 20 ones. If we consider the ROC TRUE and ROC ΔROC rules, in more than 83% of the cases the alternative chosen by the rule is one of the best 10 ones.

4.1.2 Information available about the reservation levels of the parties

In this section we considered that an alternative needs to be better than the reservation levels, for both parties, to be admissible. In our simulation study, there were cases in which there was no alternative better than the reservation levels for both parties. We eliminated these cases and increased the number of simulations in such a way to have 5000 valid examples. In Table 10 it is possible to see the average, the standard deviation and maximum number of alternatives better for both parties than the reservation levels (the minimum number was always equal to 1). Note that, using all the rules, the average number of alternatives better than the reservation levels is near 19 alternatives (in a total of 70 alternatives). In the original problem we have 26 alternatives better than the reservation levels for both parties. To determine a realistic value loss we decided to analyze only the cases in which the alternative provided by each rule is admissible. Indeed, no value loss would occur if the alternative proposed by the mediator was unacceptable for one of the parties, since there would be no agreement. As a curiosity, in Table 11 we present the proportion of cases in which each rule provides a non admissible alternative.

Table 12 presents the value loss of the different rules considering the different criteria. As expected, the worst results are obtained using ROC ROC / ROC Linear rule. Comparing with the results considering no reservation levels it is possible to see that the value loss of the criterion maximizing the sum of the values is slightly lower and the value loss of the criterion maximizing the minimal PoP is slightly higher in this case. These results are normal since we are excluding of the analysis some alternatives (the non admissible ones). Using the criterion maximizing the product of the excesses the value loss is much higher considering no reservation levels because in this case the aggregated global value is also much higher.

In Table 13, it is possible to see the proportion of cases in which the chosen alternative is efficient. Using the ROC TRUE rule the proportion is highest and using the ROC ROC / ROC Linear rule the proportion is lowest. With the criterion maximizing the minimal PoP, using the three rules, the proportion of cases in which the chosen alternative is efficient is higher considering information regarding the reservation levels. The opposite happens considering the criterion maximizing the sum of the values. In the cases in which the chosen alternative is not efficient we determined the distance to the nearest efficient alternative. The results are presented in Tables 14 and 15, using the L_2 and L_∞ distances, respectively. Considering the Euclidean distance, the best results are obtained using the ROC TRUE rule. Using the criterion maximizing the sum of the value, with all the rules, the results are better considering no reservation levels. The opposite happens when it is use the criterion maximizing de minimal PoP. With the L_∞ distance the conclusions are the same.

We computed some statistics tests to check if the difference between the ROC TRUE and ROC ΔROC rules are significative. The obtained p-values are presented in Table 16. As it is possible to see the difference between the two rules is significative considering the value loss and the criterion maximizing the minimal PoP, considering the proportion of cases in which the chosen alternative is efficient and using the criteria maximizing the sum of the values and maximizing the product of the excesses regarding the reservation level. In these cases the ROC TRUE rule presents better results. Remember that considering no reservation levels the difference between the two rules is only significative considering the proportion of cases in which the chosen alternative is efficient and using the criterion maximizing the sum of the values. It seems that the ROC ROC / ROC Linear rule is the one which provides worst results. Table 17 presents the p-values obtained when the objective was to check if the difference between the ROC ΔROC and ROC ROC / ROC Linear rules is significative. Considering the distances and the criteria maximizing the minimal PoP and maximizing the product of the excesses it is not possible to consider that the ROC Δ ROC rule is better. Remember that considering no reservation levels it is possible to conclude that the ROC Δ ROC rule is always better.

We also computed some statistics tests to compare the ROC ΔROC rule considering no reservation levels and considering reservation levels. Results are presented in Table 18. We chose this rule because between the two best rules is the one which require less information from negotiators. The difference between the two cases is significative considering the value loss and the criterion maximizing the product of the excesses, considering the proportion of cases in which the chosen alternative is efficient and the criterion maximizing the minimal PoP, and considering the L_∞ distance and the criterion maximizing the minimal PoP. In these cases better results are obtained considering reservation levels. Considering the value loss and the criterion maximizing the minimal PoP, and the distances and the criterion maximizing the sum of the values, the best results are obtained considering no reservation levels.

Tables 19 and 20 show the difference between the value of the best alternative and the value of the alternative chosen by the used criteria, for Nelson and Amstore, respectively. As it is possible to see, the rules are less prejudicial for Nelson when maximizing the product of the excesses regarding the reservation levels. Even using this criterion, in average, Nelson loses with respect to the real best alternative. The best results are obtained using the ROC TRUE rule and the worst are obtained using ROC ROC / ROC Linear rule. For Amstore, the criterion for which rules are more beneficial is to maximize the sum of the values. With this criterion, in average, Amstore wins comparing with the real best alternative. It is possible to see that, in average, both parties lose in using the rules instead of real values, except Amstore with the criterion maximizing the sum of the values. Note that considering no reservation levels the worst/best criterion for Nelson/Amstore was the same (maximizing the sum of the values) the same does not happen to the best/worst criterion for Nelson/Amstore. With these results it is also possible to consider that when the criterion maximizing the sum of the values is used, the rules does not provide a fair approximation (as it was concluded considering no reservation levels).

Results related to the position that the best alternative according to the different rules reached in the supposedly true ranking, are presented in Table 21. This table gives us the information to know how many alternatives should be chosen to guaranty the retention of the supposed best alternative. Retaining 20 alternatives the probability of retaining the best is always higher than 90%. The differences between the results considering no reservation levels and considering reservation levels are not very significative. Results relatively to the position of the best alternative using the different rules in the supposedly true ranking are shown in Table 22. In more than 90% of the cases the alternative chosen by the rule is one of the best 20 ones. If we consider the ROC TRUE and ROC ΔROC rules, in more than 85% of the cases the alternative chosen by the rule is one of the best 10 ones. The results are slightly better than the results obtained considering no reservation levels.

4.2 Examples based on the Itex vs Cypress template

4.2.1 No information available about the reservation levels of the parties

Tables 23 and 24 show the value loss of the different rules and the proportion of cases in which the chosen alternative is efficient, respectively. In the cases in which the chosen alternative is not efficient we determined the distance to the nearest efficient alternative. The results are presented in Tables 25 and 26, using the L_2 and L_∞ distances, respectively.

As it is possible to see the ROC ROC and ROC Linear rules are the ones which provide worst results, and ROC TRUE and ROC ΔROC rules are the ones which provide best results. We computed some statistics tests to check if the difference between the ROC TRUE and ROC ΔROC rules are significative. The results of the p-values are presented in Table 27. It is possible to see that the difference between the two rules is significative considering the value loss and considering the proportion of cases in which the chosen alternative is efficient. In these cases the ROC TRUE rule present better results. Note that in this example the difference between the two rules is more significative than in the example presented in section 4.1.1. The results of the statistics tests comparing the ROC ROC and ROC Linear rules are presented in Table 28. Observing the p-values it is possible to conclude that the difference between the two rules are not significative. We also compared the ROC ΔROC and ROC ROC rules (one of the best and one of the worst) to check if the difference between them are significative. In all the cases we obtained p-values equal to zero, what means that the difference between the rules is significative. So, it is possible to conclude that the ROC ΔROC rule presents better results than the ROC ROC rule. Note that, in section 4.1.1 we concluded the same.

In Tables 29 and 30, we can see the difference between the value of the best alternative and the value of the alternative chosen by the used criteria, for Itex and Cypress, respectively. As it is possible to see, the best approximation for Itex occurs when maximizing the minimal PoP, because the average difference is small. Even using this criterion, in average, Itex loses with respect to the real best alternative, because the difference is positive. The worst criterion for Itex is maximizing the sum of the values. For Cypress, the best criterion is to maximize the sum of the values but, even with this criterion, in average, Cypress loses comparing with the real best alternative. For Cypress, the worst results are obtained using the criterion maximizing the product of the excesses regarding the reservation levels. It is possible to see that, in average, both parties loses in using the rules instead of real values. With these results it is also possible to see if the alternative chosen by the rule is a fair approximation. As we concluded in subsection

4.1.1., it is possible to consider that using the criterion maximizing the sum of the values, the rules do not provide a fair approximation.

Results related to the position that the best alternative according to the different rules reached in the supposedly true ranking, are presented in Table 31. As expected, as the total number of alternatives is equal to 180, retaining only one alternative is not sufficient in the majority of the cases. However, even then, we can consider that the hit rate, is quite reasonable. Retaining 20 alternatives (close to 12% of the total number of alternatives) the probability of retaining the best is higher than 85%, considering the ROC TRUE and ROC ΔROC rules. Retaining 20 alternatives in this example correspond, more or less, to retaining 10 alternatives in the example presented in the last section. The obtained results are not very different. Results relatively to the position of the best alternative using the different rule in the supposedly true ranking are shown in Table 32. In more than 72% of the cases the alternative chosen by the rule is one of the best 20 ones. If we consider the ROC TRUE and ROC ΔROC rules, in more than 74% of the cases the alternative chosen by the rule is one of the best 10 ones. Once more, the obtained results are not very different from the ones obtained in subsection 4.1.1.

4.2.2 Information available about the reservation levels of the parties

In Table 33, it is possible to see the average, the standard deviation and maximum number of alternatives better than the reservation levels for both parties, considering that the reservation level of Itex is equal to $V^I(x_{86})$ and the reservation level of Cypress is equal to $V^C(x_{95})$. Note that, using all the rules, the average number of alternatives better than the reservation levels is near 38 alternatives (in a total of 180 alternatives). We will not present the results obtained considering that the reservation levels are equal to the values of the alternatives 52 and 129, for Itex and Cypress, respectively, because they are not very different from the ones obtained considering that there are no reservation levels.

We can see in Table 34 the proportion of cases in each the alternative chosen by each rule is not admissible. Table 35 shows the value loss of the different rules considering the different criteria and Table 36 shows the proportion of cases in which the chosen alternative is efficient. In the cases in which the chosen alternative is not efficient we determined the distance to the nearest efficient alternative. The results are presented in Tables 37 and 38, using the L_2 and L_∞ distances, respectively.

We performed some statistical tests to check if the difference between the ROC TRUE values and ROC ΔROC rules (the two best rules) are significative. The results of the p-values are presented in Table 39. The only cases where it is not possible to conclude that the ROC TRUE rule is better is considering the distances and the criterion maximizing the product of the excesses regarding the reservation levels. Note that the difference between the two rules is more significative considering reservation levels and more significative than in section 4.1.2. The ROC ROC and ROC Linear rules are the ones which provide worst results, so we computed some statistics tests to see if the difference between them is significative. The results are presented in Table 40. In all the cases it is possible to conclude that the two rules present similar results (as it was concluded considering no reservation levels). Similarly to what we done in the previous subsection we also compared the ROC ΔROC and ROC ROC rules. In all the cases we obtained p-values equal to zero, or very close, what means that the difference between the two rules is significative, this difference is favorable to the ROC ΔROC rule. This is the same conclusion we obtained considering no reservation levels. Remember that in the examples for the Nelson vs. Amstore template (see section 4.1.2) it was not always possible to conclude that the ROC ΔROC rule is better.

As in the examples for the Nelson vs. Amstore template, we compared the ROC ΔROC rule considering reservation values and considering no reservation levels (see results in Table 41). Considering the value loss and the criteria maximizing the sum of the values and maximizing the product of the excesses the best results are obtained considering reservation levels. Considering the value loss and the criterion maximizing the minimal PoP, the proportion of cases in which the chosen alternative is efficient and the criteria maximizing the sum of the values and maximizing the minimal PoP, the distances and the criterion maximizing the product of the excesses the best results are obtained considering no reservation levels. The results are very different from the ones obtained in subsection 4.1.2. However this difference is natural since in one example we are considering reservation levels very different between the parties and in the other example the reservation levels of both parties are similar.

In Tables 42 and 43, it is possible to see the difference between the value of the best alternative and the value of the alternative chosen by the used criteria, for Itex and Cypress, respectively. The rules provide a better approximation for Itex when maximizing the minimal PoP. Even using this criterion, in average, Itex loses with respect to the real best alternative. For Cypress, the best approximation occurs for the criterion maximize the sum of the values but even with this criterion, in average, Cypress loses comparing with the real best alternative. The conclusions are the same obtained considering no reservation levels.

Table 44 presents the position that the best alternative according to the different rules reached in the supposedly true ranking. Results relatively to the position of the best alternative using the different rule in the supposedly true ranking are shown in Table 45. The results are not very different from the ones considering no reservation levels.

5 Conclusions

In bilateral Negotiation Analysis, the literature often considers the case with complete information, which cannot be applied in practice when the parties do not have complete information. In this work we considered a setting of bilateral integrative negotiation over multiple issues in which the preferences of both parties can be roughly modelled by an additive value function. We compared four decision rules (ROC TRUE, ROC ΔROC, ROC ROC and ROC Linear) to help a mediator suggesting an alternative under these circumstances, considering that there exists ordinal information both on the scaling weights and on the level' values, and tested them using Monte-Carlo simulation.

We compared the four rules using two cases as templates for generating random examples (one with 70 alternatives and the other one with 180 alternatives). In all cases we used three criteria: maximizing the sum of the values, maximizing the product of the excesses regarding the reservation levels and maximizing the minimal PoP. Note however that our objective was not to compare the criteria but to compare the rules. The choice of the used criteria depends on the mediator preferences.

To compare the rules we determined the proportion of cases in which each rule chose the really best alternatives and in the cases in which the two rules did not match we determined the value loss. We determined the proportion of cases in which the alternative chosen by each rule is efficient, and for the inefficient ones we determined the distance to the nearest efficient alternative (using the L_2 and L_∞ distances). The results using the two cases as templates for generating random examples were different and the results considering no reservation levels and considering reservation levels were also different. But, it is possible to conclude that the best results were obtained using the ROC TRUE and ROC ΔROC rules. There are cases in which it is possible to consider that these rules are equivalent which is, in a certain way, surprising, because in the ROC TRUE rule we require cardinal information about the levels' values from the negotiators. The ROC ROC and ROC Linear rules are the worst ones, and it is possible to consider that the difference between them is not significative. The ROC ΔROC rule is, almost always, better than the ROC ROC rule. We determined also the difference between the real value of the best alternative and the real value of the alternative chosen by the used criterion, for both parties. This enables us to know whether using only ordinal information leads to treat one of the parties unfairly, when compared to a situation in which precise cardinal values were used instead. We also compared the ranking of the alternatives according to the supposedly true parameters with the ranking of the alternatives according to the used decision rule, we considered the following results: the position that the best alternative according to the true ranking reaches in the ranking generated by the used decision rule (this allows us to know the minimum number of alternatives that must be chosen, beginning by the top of the ranking provided by the rule, so that the true best alternative belongs to the chosen set) and the position that the best alternative in the ranking generated by the rule reaches in the supposedly true ranking (this allows us to know how good the alternative chosen by the rule is in terms of the supposed true ranking).

In our opinion the results are encouraging. Considering that the total number of alternatives is high the hit rate is relatively good and when the rule does not choose the real true alternative the average value loss is not very high. The proportion of cases in which the chosen alternative is efficient is high and when the chosen alternative is not efficient the distance to the nearest efficient alternative is not very high. The position that the best alternative in the ranking generated by the rule reaches in the supposedly true ranking complements that results that show the alternative chosen by the rule is typically a good alternative. In the majority of the cases negotiators, in average, lose in using the rules instead of real values for all the criteria, however the average loss is not very high. The criterion maximizing the sum of the values is the one which provides less balanced approximations between the two parties. It was possible to see that, although the hit rate is relatively high, retaining one alternative is not sufficient in the majority of the cases. But, if instead of one alternative, the mediator keeps for example a set of 20 alternatives, the probability of this set containing the real best alternative is very high. The position that the best alternative according to the true ranking reaches in the ranking generated by the used decision rule allowed us to assess the strategy of retaining $k < m$ alternatives instead of only one.

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A Results

Sum

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	9.1331	10.2414	82.7968	12.7696	30.3641
ROC ΔROC	9.3535	10.1638	74.8539	12.7093	29.7159
ROC ROC = ROC Linear	14.8723	13.3471	91.6100	21.4859	41.6842

Product

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	666.6783	758.1803	5064.9614	942.2391	2249.4214
ROC ΔROC	656.0594	758.7748	5452.6670	884.8055	2233.3544
ROC ROC = ROC Linear	925.0817	847.7156	6665.5542	1319.7663	2622.4553

PoP

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	0.0819	0.0772	0.5393	0.1182	0.2377
ROC ΔROC	0.0857	0.0802	0.5808	0.1198	0.2477
ROC ROC = ROC Linear	0.1140	0.1015	0.8017	0.1639	0.3179

Table 1: Value Loss (reservation levels equal to 0 for Nelson and Amstore).

	Sum	Product	PoP
ROC TRUE	94.26	91.06	80.92
ROC ΔROC	92.22	90.22	79.18
ROC ROC = ROC Linear	88.72	78.36	67.46

Table 2: Proportion of cases in which the chosen alternative is efficient (reservation levels equal to 0 for Nelson and Amstore).

Sum

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	6.3796	4.9173	26.3772	8.5454	15.9898
ROC ΔROC	6.0499	4.8757	38.7505	8.1107	15.4725
ROC ROC = ROC Linear	7.5029	5.9274	36.7852	10.3016	19.4668

Product

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	7.9482	6.2996	42.2776	10.1963	21.1535
ROC ΔROC	8.0421	6.4076	36.5928	10.5669	20.9423
ROC ROC = ROC Linear	10.6254	9.5886	58.8975	14.1004	30.6239

PoP

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	9.5344	7.1672	42.0696	12.6078	24.4005
ROC ΔROC	9.5435	7.4248	42.6280	13.1596	24.7427
ROC ROC = ROC Linear	11.0923	8.9947	58.8975	14.8755	29.6205

Table 3: Distance between the chosen alternative and the nearest efficient alternative - Euclidean distance (reservation levels equal to 0 for Nelson and Amstore).

Sum

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	5.5997	4.3370	25.9145	7.5848	14.2417
ROC ΔROC	5.2915	4.2197	34.3092	7.1573	13.0022
ROC ROC = ROC Linear	6.5896	5.1403	33.4238	9.1746	16.2932

Product

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	7.0156	5.5839	33.1487	9.2238	18.7618
ROC ΔROC	7.1175	5.7924	33.3152	9.2655	19.1413
ROC ROC = ROC Linear	9.2627	8.2360	53.4606	12.4060	26.6948

PoP

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	8.5284	6.4665	38.2792	11.5202	21.6739
ROC ΔROC	8.5705	6.8003	39.5629	11.9348	22.3459
ROC ROC = ROC Linear	9.8039	7.8970	53.4606	13.3540	25.8158

Table 4: Distance between the chosen alternative and the nearest efficient alternative - L_∞ distance (reservation levels equal to 0 for Nelson and Amstore).

	Sum	Product	PoP
Value Loss	0.4037	0.5889	0.0390
Efficient	0.0001	0.1493	0.0294
Euclidean Distance	0.3872	0.8213	0.9778
L_∞ Distance	0.3556	0.7841	0.8873

Table 5: p - values comparing the ROC TRUE and ROC ΔROC rules (reservation levels equal to 0 for Nelson and Amstore).

	Sum	Product	PoP
ROC TRUE	8.3781	3.4446	3.2014
ROC ΔROC	8.8379	4.0453	3.4283
ROC ROC = ROC Linear	20.7354	6.0176	4.5321

Table 6: Average of the difference between the value of the best alternative and the value of the alternative chosen by the used criterion, for Nelson (reservation levels equal to 0 for Nelson and Amstore).

	Sum	Product	PoP
ROC TRUE	-3.1522	1.8095	3.2114
ROC ΔROC	-2.9489	1.8195	2.8835
ROC ROC = ROC Linear	-9.4563	5.0615	5.9737

Table 7: Average of the difference between the value of the best alternative and the value of the alternative chosen by the used criterion, for Amstore (reservation levels equal to 0 for Nelson and Amstore).

Sum

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	4.1774	53.68	64.72	73.14	76.32	81.68	89.84	95.84
ROC ΔROC	4.8620	49.02	61.20	70.02	74.48	78.94	87.88	94.22
ROC Linear = ROC ROC	6.8776	56.26	60.68	64.32	67.44	70.54	81.32	88.28

Product

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	3.5760	48.24	62.56	71.20	77.06	81.70	92.82	98.16
ROC ΔROC	4.1648	41.14	56.20	65.54	72.64	77.64	90.48	97.58
ROC Linear = ROC ROC	7.5086	26.72	36.98	42.88	53.38	57.30	76.80	89.70

PoP

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	4.7068	28.36	45.52	54.52	62.78	68.92	89.80	98.42
ROC ΔROC	5.1068	25.70	42.22	51.58	59.38	66.30	87.54	98.00
ROC Linear = ROC ROC	6.9452	20.00	33.18	40.74	47.44	55.34	78.44	93.80

Table 8: Position of the supposedly best alternative in the ranking induced by the different rules (reservation levels equal to 0 for Nelson and Amstore).

Sum

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	5.6654	43.06	57.40	65.72	71.78	75.46	85.90	92.80
ROC ΔROC	6.1456	37.04	51.96	60.72	67.04	71.84	83.84	92.54
ROC Linear = ROC ROC	11.4146	26.42	37.86	44.68	50.18	54.54	68.98	81.54

Product

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	4.6790	43.74	57.70	67.78	74.04	78.38	88.06	94.64
ROC ΔROC	5.0938	36.66	53.12	63.76	70.10	75.50	87.26	94.24
ROC Linear = ROC ROC	8.7072	22.50	33.50	42.82	51.10	56.78	74.16	88.10

PoP

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	5.2058	28.36	40.82	52.98	62.32	69.32	87.14	96.58
ROC ΔROC	5.4474	25.46	37.46	50.58	60.94	67.94	86.12	96.40
ROC Linear = ROC ROC	7.4214	17.02	27.60	40.12	49.46	56.28	78.20	92.42

Table 9: Position of the best alternative according to the different rules in the supposedly true ranking (reservation levels equal to 0 for Nelson and Amstore).

	Average	Std Deviation	Maximum
ROC TRUE	17.6903	12.0926	62
ROC ΔROC	17.7689	12.5636	64
ROC ROC = ROC Linear	19.0303	13.6112	64

Table 10: Number of alternatives better than the reservation levels for both parties (reservation levels equal to $V^N(x_{25})$ and $V^A(x_{64})$ for Nelson and Amstore, respectively).

	Sum	Product	PoP
ROC TRUE	0.1489	0.1052	0.0947
ROC ΔROC	0.1775	0.1214	0.1128
ROC ROC / ROC Linear	0.2362	0.1728	0.1531

Table 11: Proportion of cases in which the alternative chosen by the rule is not admissible (reservation levels equal to $V^N(x_{25})$ and $V^A(x_{64})$ for Nelson and Amstore, respectively).

Sum

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	8.5090	10.4751	85.9244	11.6505	29.0580
ROC ΔROC	8.6598	10.1872	70.0000	11.6663	29.7934
ROC ROC = ROC Linear	10.7544	11.1101	89.2211	14.9577	33.7866

Product

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	208.6108	304.1034	2617.3030	255.2163	776.7140
ROC ΔROC	210.6686	277.6787	2526.7637	277.0149	762.7409
ROC ROC = ROC Linear	319.3760	387.0792	4993.2212	435.3348	1075.5727

PoP

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	0.1681	0.1645	0.9362	0.2362	0.5048
ROC ΔROC	0.1797	0.1679	0.9740	0.2621	0.5152
ROC ROC = ROC Linear	0.2210	0.1912	0.9784	0.3319	0.6031

Table 12: Value Loss (reservation levels equal to $V^N(x_{25})$ and $V^A(x_{64})$ for Nelson and Amstore, respectively).

	Sum	Product	PoP
ROC TRUE	92.36	91.24	85.60
ROC ΔROC	89.60	89.20	84.02
ROC ROC = ROC Linear	78.13	78.13	71.90

Table 13: Proportion of cases in which the chosen alternative is efficient (reservation levels equal to $V^N(x_{25})$ and $V^A(x_{64})$ for Nelson and Amstore, respectively).

Sum

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	7.4556	5.3827	35.4586	10.0752	17.1159
ROC ΔROC	7.3184	5.9418	35.9007	10.2493	19.3874
ROC ROC = ROC Linear	8.9726	7.5034	51.2293	12.4509	24.6487

Product

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	8.3894	5.9185	35.0713	11.9186	19.2113
ROC ΔROC	8.7418	6.9838	39.4867	11.7945	22.3840
ROC ROC = ROC Linear	9.6347	7.7069	52.1393	13.4973	25.2543

PoP

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	8.0832	5.8025	32.0241	11.1472	19.1810
ROC ΔROC	8.6453	7.0068	39.4867	11.9391	22.6442
ROC ROC = ROC Linear	9.5111	7.6036	52.1393	13.2749	25.1272

Table 14: Distance between the chosen alternative and the nearest efficient alternative - Euclidean distance (reservation levels equal to $V^N(x_{25})$ and $V^A(x_{64})$ for Nelson and Amstore, respectively).

Sum

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	6.6105	4.8314	29.6134	8.8616	15.6956
ROC ΔROC	6.4633	5.2190	29.3920	9.0571	17.1782
ROC ROC = ROC Linear	7.8849	6.5620	49.2881	10.9297	21.7608

Product

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	7.4049	5.3039	29.5458	10.3116	17.0891
ROC ΔROC	7.6524	6.1198	32.3248	10.2590	19.8609
ROC ROC = ROC Linear	8.4602	6.6965	45.5016	11.4770	21.7481

PoP

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	7.2076	5.2186	29.5458	10.0637	16.9583
ROC ΔROC	7.7054	6.3278	34.2648	10.3447	20.0945
ROC ROC = ROC Linear	8.4078	6.6624	45.5016	11.4281	22.5617

Table 15: Distance between the chosen alternative and the nearest efficient alternative - L_∞ distance (reservation levels equal to $V^N(x_{25})$ and $V^A(x_{64})$ for Nelson and Amstore, respectively).

	Sum	Product	PoP
Value Loss	0.6403	0.8159	0.0094
Efficient	0.0000	0.0024	0.0707
Euclidean Distance	0.7415	0.4362	0.1194
L_∞ Distance	0.5499	0.5369	0.1261

Table 16: p - values comparing the ROC TRUE and ROC ΔROC rules (reservation levels equal to $V^N(x_{25})$ and $V^A(x_{64})$ for Nelson and Amstore, respectively).

	Sum	Product	PoP
Value Loss	0.0000	0.0000	0.0000
Efficient	0.0000	0.0000	0.0000
Euclidean Distance	0.0000	0.0275	0.0112
L_∞ Distance	0.0010	0.0225	0.0213

Table 17: p - values comparing the ROC ROC and ROC ΔROC rules (reservation levels equal to $V^N(x_{25})$ and $V^A(x_{64})$ for Nelson and Amstore, respectively).

	Sum	Product	PoP
Value Loss	0.0150	0.0000	0.0000
Efficient	0.2312	0.0742	0.0000
Euclidean Distance	0.0008	0.1113	0.0112
L_∞ Distance	0.0000	0.1706	0.0076

Table 18: p - values comparing the ROC ΔROC and ROC ΔROC rules considering no reservation levels and considering reservation levels (reservation levels equal to $V^N(x_{25})$ and $V^A(x_{64})$ for Nelson and Amstore, respectively).

	Sum	Product	PoP
ROC TRUE	5.0279	1.1744	1.5013
ROC ΔROC	5.6932	1.7034	1.8331
ROC ROC = ROC Linear	10.6905	3.3974	3.7524

Table 19: Average of the difference between the value of the best alternative and the value of the alternative chosen by the used criterion, for Nelson (reservation levels equal to $V^N(x_{25})$ and $V^A(x_{64})$ for Nelson and Amstore, respectively).

	Sum	Product	PoP
ROC TRUE	-0.7197	2.5724	2.2709
ROC ΔROC	-0.2822	2.9296	2.3836
ROC ROC = ROC Linear	-1.1040	5.4362	4.3285

Table 20: Average of the difference between the value of the best alternative and the value of the alternative chosen by the used criterion, for Amstore (reservation levels equal to $V^N(x_{25})$ and $V^A(x_{64})$ for Nelson and Amstore, respectively).

Sum

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	3.9941	46.95	60.52	69.38	75.39	79.64	90.31	97.39
ROC ΔROC	4.4883	42.21	56.08	66.04	71.66	76.66	88.58	96.43
ROC Linear = ROC ROC	5.8016	43.12	51.65	57.79	63.38	67.66	82.57	93.54

Product

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	3.3509	46.17	62.61	72.19	78.41	82.97	93.78	99.06
ROC ΔROC	3.7519	40.66	58.26	68.39	75.03	79.93	92.12	98.69
ROC Linear = ROC ROC	5.5518	27.05	43.38	53.96	61.07	67.19	84.09	96.51

PoP

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	3.9202	33.07	51.12	63.03	71.92	77.54	92.90	99.15
ROC ΔROC	4.2970	30.22	48.34	59.96	69.07	75.20	90.97	98.55
ROC Linear = ROC ROC	5.7918	21.00	36.99	48.18	57.77	64.20	83.86	96.72

Table 21: Position of the supposedly best alternative in the ranking induced by the different rules (reservation levels equal to $V^N(x_{25})$ and $V^A(x_{64})$ for Nelson and Amstore, respectively).

Sum

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	4.6979	42.03	56.70	65.61	72.17	76.24	86.50	96.37
ROC ΔROC	5.1184	35.90	51.24	61.49	67.96	72.68	85.13	96.06
ROC Linear = ROC ROC	7.6877	24.60	36.94	46.38	52.89	58.12	73.54	90.86

Product

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	3.4862	46.17	60.68	70.88	77.02	81.51	92.98	99.16
ROC ΔROC	3.8166	40.66	56.12	67.00	74.60	79.31	91.73	99.02
ROC Linear = ROC ROC	5.5110	27.05	41.40	52.34	60.10	66.50	83.16	97.30

PoP

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	3.9925	33.07	49.40	61.55	70.97	76.49	92.66	99.39
ROC ΔROC	4.2480	30.22	46.40	59.12	68.73	75.10	91.34	99.15
ROC Linear = ROC ROC	5.7214	21.00	35.58	46.97	55.74	63.07	83.64	97.78

Table 22: Position of the best alternative according to the different rules in the supposedly true ranking (reservation levels equal to $V^N(x_{25})$ and $V^A(x_{64})$ for Nelson and Amstore, respectively).

Sum

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	6.5015	7.8848	81.9559	8.6773	21.5471
ROC ΔROC	8.0700	8.5475	76.4573	11.2015	24.0444
ROC ROC	13.8244	12.8242	92.7779	19.4077	39.6037
ROC Linear	13.6954	12.7268	107.4878	19.0680	38.0641

Product

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	495.8791	592.5318	5534.3892	662.4846	1644.0372
ROC ΔROC	566.6768	574.4958	4992.4771	791.1349	1702.3535
ROC ROC	901.5501	763.2006	4940.0244	1311.0825	2391.2342
ROC Linear	942.3477	815.1173	7282.7578	1361.1470	2499.7881

PoP

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	0.0613	0.0586	0.6000	0.0865	0.1758
ROC ΔROC	0.0713	0.0624	0.5090	0.1036	0.1905
ROC ROC	0.1036	0.0840	0.5717	0.1486	0.2647
ROC Linear	0.1026	0.0855	0.6106	0.1471	0.2684

Table 23: Value Loss (reservation levels equal to 0 for Itex and Cypress).

	Sum	Product	PoP
ROC TRUE	93.52	91.52	78.08
ROC ΔROC	86.70	83.92	71.58
ROC ROC	70.76	64.28	52.24
ROC Linear	68.86	64.42	54.38

Table 24: Proportion of cases in which the chosen alternative is efficient (reservation levels equal to 0 for Itex and Cypress).

Sum

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	4.5382	3.6693	31.3354	5.9837	10.6441
ROC ΔROC	5.1697	4.1178	31.6802	7.0169	13.0660
ROC ROC	7.9413	6.6948	41.9947	10.7435	21.3565
ROC Linear	7.6789	6.4491	46.6887	10.2455	21.1384

Product

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	5.8982	5.2021	37.2921	7.6332	15.8221
ROC ΔROC	5.4954	4.4734	32.1554	7.6322	14.5 084
ROC ROC	8.4742	7.3383	49.2268	11.5466	23.4395
ROC Linear	8.3068	6.9107	44.0468	11.2836	22.8509

PoP

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	6.9010	6.0906	46.5559	8.8486	20.0976
ROC ΔROC	6.9352	5.8390	43.2304	8.9151	18.5517
ROC ROC	8.8898	7.1920	49.4757	12.2976	23.1018
ROC Linear	9.0281	7.1493	54.3297	12.4337	23.0572

Table 25: Distance between the chosen alternative and the nearest efficient alternative - Euclidean distance (reservation levels equal to 0 for Itex and Cypress).

Sum

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	4.0128	3.3228	31.2611	5.1972	9.4800
ROC ΔROC	4.5543	3.5643	24.8643	6.2533	11.0313
ROC ROC	6.9497	5.7741	40.5663	9.3560	18.9336
ROC Linear	6.7018	5.5550	42.9326	8.9152	18.1948

Product

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	5.1963	4.5181	31.6275	6.5837	13.9875
ROC ΔROC	4.8280	3.8895	26.2871	6.5516	12.7212
ROC ROC	7.3850	6.3283	41.7894	10.0931	19.8200
ROC Linear	7.2085	5.8847	40.2117	9.5947	19.2210

PoP

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	6.1102	5.3582	46.3592	7.8851	17.4627
ROC ΔROC	6.1127	5.0719	35.4446	7.9285	16.6225
ROC ROC	7.7702	6.2279	41.7894	10.7844	19.9629
ROC Linear	7.8656	6.1417	47.2825	10.6376	19.5373

Table 26: Distance between the chosen alternative and the nearest efficient alternative - L_∞ distance (reservation levels equal to 0 for Itex and Cypress).

	Sum	Product	PoP
Value Loss	0.0000	0.0000	0.0000
Efficient	0.0000	0.0000	0.0000
Euclidean Distance	0.0147	0.1763	0.8869
L_∞ Distance	0.0189	0.1547	0.9905

Table 27: p - values comparing the ROC TRUE and ROC ΔROC rules (reservation levels equal to 0 for Itex and Cypress).

	Sum	Product	PoP
Value Loss	0.6465	0.0194	0.5820
Efficient	0.0385	0.8838	0.0319
Euclidean Distance	0.2733	0.4832	0.5100
L_∞ Distance	0.2298	0.3885	0.5946

Table 28: p - values comparing the ROC ROC and ROC Linear rules (reservation levels equal to 0 for Itex and Cypress).

	Sum	Product	PoP
ROC TRUE	3.5428	2.0298	1.7836
ROC ΔROC	4.8839	2.4150	1.8617
ROC ROC	11.2240	5.4766	4.3485
ROC Linear	8.7135	5.6907	4.2226

Table 29: Average of the difference between the value of the best alternative and the value of the alternative chosen by the used criterion, for Itex (reservation levels equal to 0 for Itex and Cypress).

	Sum	Product	PoP
ROC TRUE	0.3971	1.8884	1.6044
ROC ΔROC	0.7679	2.9493	2.2982
ROC ROC	0.2227	5.4766	4.3516
ROC Linear	2.5497	5.2677	4.1894

Table 30: Average of the difference between the value of the best alternative and the value of the alternative chosen by the used criterion, for Cypress (reservation levels equal to 0 for Itex and Cypress).

Sum

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	5.1624	47.52	60.64	70.56	75.00	79.72	88.28	94.80
ROC ΔROC	6.8662	43.52	53.86	61.26	67.52	71.50	82.76	91.76
ROC ROC	14.3946	37.34	41.66	46.22	49.44	53.66	66.06	77.54
ROC Linear	18.0544	25.78	31.88	37.22	43.82	46.56	60.38	74.84

Product

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	5.1538	42.40	56.76	65.90	71.58	75.98	87.88	95.06
ROC ΔROC	7.3708	31.86	46.40	55.54	61.64	66.46	80.32	90.98
ROC ROC	15.4054	18.44	26.18	32.66	38.28	42.58	59.14	75.96
ROC Linear	15.4960	19.04	27.42	33.40	38.96	43.42	58.64	75.92

PoP

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	7.5956	25.36	39.20	48.58	56.06	61.88	78.00	90.46
ROC ΔROC	9.8108	21.58	32.24	40.28	47.72	52.98	70.00	85.10
ROC ROC	17.7646	12.52	19.18	24.24	29.34	33.78	51.00	69.56
ROC Linear	17.5306	14.04	20.74	26.76	31.10	36.00	51.54	70.80

Table 31: Position of the supposedly best alternative in the ranking induced by the different rules (reservation levels equal to 0 for Itex and Cypress).

Sum

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	6.2516	39.40	55.64	64.56	70.16	74.56	85.54	93.30
ROC ΔROC	9.1728	29.94	43.64	52.96	59.76	64.42	78.22	88.64
ROC ROC	20.8542	17.20	27.16	33.52	38.82	43.26	57.88	72.88
ROC Linear	19.8978	17.76	26.98	33.94	39.02	43.84	58.76	74.22

Product

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	5.7854	41.64	55.52	65.18	71.16	75.42	86.72	93.98
ROC ΔROC	7.5408	31.24	45.84	54.58	61.44	66.62	80.86	90.98
ROC ROC	16.4602	17.88	28.04	35.26	40.36	45.36	60.52	76.00
ROC Linear	16.3464	18.36	27.52	34.60	40.34	45.60	61.18	76.08

PoP

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	6.9946	25.36	39.90	49.08	56.72	62.40	80.86	93.08
ROC ΔROC	8.7976	21.48	33.94	42.62	50.12	56.08	74.52	88.98
ROC ROC	15.1948	12.20	20.68	27.52	33.54	38.12	56.96	76.20
ROC Linear	15.0010	13.64	22.34	29.78	35.50	40.58	58.10	76.74

Table 32: Position of the best alternative according to the different rules in the supposedly true ranking (reservation levels equal to 0 for Itex and Cypress).

	Average	Std Deviation	Maximum	Minimum
ROC TRUE	38.2726	19.1917	110	2
ROC ΔROC	38.1759	17.4110	97	2
ROC ROC	37.2910	12.4327	52	4
ROC Linear	36.7956	13.0861	55	2

Table 33: Number of alternatives better than the reservation levels for both parties (reservation levels equal to $V^I(x_{86})$ and $V^C(x_{95})$ for Itex and Cypress, respectively).

	Sum	Product	PoP
ROC TRUE	0.1254	0.0494	0.0495
ROC ΔROC	0.1510	0.0790	0.0790
ROC ROC	0.2111	0.1215	0.1264
ROC Linear	0.2078	0.1236	0.1237

Table 34: Proportion of cases in which the alternative chosen by the rule is not admissible (reservation levels equal to $V^I(x_{86})$ and $V^C(x_{95})$ for Itex and Cypress, respectively).

Sum

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	5.9463	6.5508	54.0802	8.1745	19.1710
ROC ΔROC	6.8367	6.9591	47.0883	9.5945	20.8384
ROC ROC	10.1607	8.9225	59.6517	14.8223	28.1451
ROC Linear	9.9807	8.8235	71.6798	14.2356	27.3813

Product

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	172.2259	214.5850	1797.9627	226.5396	617.2684
ROC ΔROC	199.1753	218.5182	1501.9280	271.1409	659.2973
ROC ROC	294.5875	263.9202	1641.3989	428.1743	821.5392
ROC Linear	300.3229	264.4241	1608.4762	426.5116	815.7595

PoP

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	0.1598	0.1441	0.8820	0.2298	0.4486
ROC ΔROC	0.1738	0.1508	0.8708	0.2532	0.4807
ROC ROC	0,2123	0,1661	0,8774	0,3156	0,5331
ROC Linear	0.2065	0.1621	0.8464	0.3002	0.5272

Table 35: Value Loss (reservation levels equal to $V^I(x_{86})$ and $V^C(x_{95})$ for Itex and Cypress, respectively).

	Sum	Product	PoP
ROC TRUE	90.91	88.47	79.79
ROC ΔROC	84.28	80.84	71.13
ROC ROC	68.69	58.17	54.20
ROC Linear	68.38	60.48	56.50

Table 36: Proportion of cases in which the chosen alternative is efficient (reservation levels equal to $V^I(x_{86})$ and $V^C(x_{95})$ for Itex and Cypress, respectively).

Sum

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	4.8535	4.0202	25.2434	6.3320	13.2327
ROC ΔROC	5.6929	4.6677	31.8151	7.6128	15.0990
ROC ROC	8.8975	7.8206	49.3784	11.9883	24.2859
ROC Linear	8.3629	7.0988	51.5835	11.3329	23.0141

Product

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	5.6735	4.7981	35.0469	7.2301	14.7303
ROC ΔROC	6.1966	4.9340	31.0535	8.1386	16.5560
ROC ROC	8.4434	6.9001	57.2317	11.3165	22.6750
ROC Linear	8.7796	7.6751	70.3028	11.7894	24.861

PoP

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	6.2399	5.0706	35.0469	8.1630	17.0049
ROC ΔROC	6.8458	5.3402	39.1335	9.2583	17.3928
ROC ROC	8.5837	6.8962	57.2317	11.8020	22.3722
ROC Linear	8.9705	7.5280	70.3028	12.4112	23.4569

Table 37: Distance between the chosen alternative and the nearest efficient alternative - Euclidean distance (reservation levels equal to $V^I(x_{86})$ and $V^C(x_{95})$ for Itex and Cypress, respectively).

Sum

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	4.3276	3.6287	24.6506	5.5945	11.2268
ROC ΔROC	4.9989	4.0315	26.3850	6.5698	13.7558
ROC ROC	7.6887	6.5601	41.3378	10.4096	20.4455
ROC Linear	7.2723	6.0841	44.4360	9.8583	19.9724

Product

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	5.0177	4.1917	27.2377	6.5673	12.9298
ROC ΔROC	5.4665	4.3637	24.5040	7.2484	14.6581
ROC ROC	7.3520	5.9657	57.2285	9.8184	19.5012
ROC Linear	7.6315	6.5354	52.7250	10.3646	20.5019

PoP

	Average	Std Deviation	Maximum	P_{75}	P_{95}
ROC TRUE	5.5247	4.4333	29.3216	7.3052	14.3647
ROC ΔROC	6.0238	4.6960	32.9987	8.1621	15.5254
ROC ROC	7.4665	5.9658	57.2285	10.1244	19.2476
ROC Linear	7.8041	6.4114	52.7250	10.8046	20.1950

Table 38: Distance between the chosen alternative and the nearest efficient alternative - L_∞ distance (reservation levels equal to $V^I(x_{86})$ and $V^C(x_{95})$ for Itex and Cypress, respectively).

	Sum	Product	PoP
Value Loss	0.0000	0.0000	0.0000
Efficient	0.0000	0.0000	0.0000
Euclidean Distance	0.0008	0.0401	0.0043
L_∞ Distance	0.0025	0.0448	0.0073

Table 39: p - values comparing the ROC TRUE and ROC ΔROC rules (reservation levels equal to $V^I(x_{86})$ and $V^C(x_{95})$ for Itex and Cypress, respectively).

	Sum	Product	PoP
Value Loss	0.4275	0.3648	0.1341
Efficient	0.7356	0.0182	0.0203
Euclidean Distance	0.0439	0.1410	0.0732
L_∞ Distance	0.0639	0.1534	0.0682

Table 40: p - values comparing the ROC ROC and ROC Linear rules (reservation levels equal to $V^I(x_{86})$ and $V^C(x_{95})$ for Itex and Cypress, respectively).

	Sum	Product	PoP
Value Loss	0.0000	0.0000	0.0000
Efficient	0.0006	0.0001	0.6185
Euclidean Distance	0.0232	0.0017	0.6689
L_∞ Distance	0.0257	0.0012	0.6263

Table 41: p - values comparing the ROC ΔROC and ROC ΔROC rules considering reservation levels and considering no reservation levels (reservation levels equal to $V^I(x_{86})$ and $V^C(x_{95})$ for Itex and Cypress, respectively).

	Sum	Product	PoP
ROC TRUE	2.3057	1.7337	1.5767
ROC ΔROC	3.7951	2.6102	2.3648
ROC ROC	8.1129	5.2274	4.2974
ROC Linear	6.7649	5.5292	4.3161

Table 42: Average of the difference between the value of the best alternative and the value of the alternative chosen by the used criterion, for Itex (reservation levels equal to $V^I(x_{86})$ and $V^C(x_{95})$ for Itex and Cypress, respectively).

	Sum	Product	PoP
ROC TRUE	1.2421	1.9209	1.7385
ROC ΔROC	1.4292	2.6682	2.4730
ROC ROC	2.5780	5.3033	4.3695
ROC Linear	3.6710	5.1048	4.3930

Table 43: Average of the difference between the value of the best alternative and the value of the alternative chosen by the used criterion, for Cypress (reservation levels equal to $V^I(x_{86})$ and $V^C(x_{95})$ for Itex and Cypress, respectively).

Sum

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	6.6694	42.63	56.28	64.00	68.47	72.79	82.60	90.22
ROC ΔROC	8.1518	37.94	48.20	55.59	61.61	66.21	78.04	87.15
ROC ROC	11.2434	33.88	38.15	42.28	46.53	51.03	63.66	81.02
ROC Linear	11.1575	25.52	31.88	37.34	44.25	47.82	64.86	83.25

Product

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	5.3612	40.35	54.32	62.91	68.70	73.13	84.58	94.88
ROC ΔROC	7.1152	29.09	42.52	51.05	57.42	62.58	78.48	91.41
ROC ROC	12.2043	19.16	25.71	32.41	36.46	41.68	57.98	79.09
ROC Linear	11.3907	18.51	25.85	32.58	37.58	41.85	61.09	82.66

PoP

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	6.3178	27.36	41.46	50.72	58.68	64.18	81.32	94.11
ROC ΔROC	8.0142	21.60	33.33	41.92	48.52	54.99	74.19	90.37
ROC ROC	12.9865	14.36	21.01	27.10	31.97	37.61	55.84	77.34
ROC Linear	12.0989	14.80	22.44	28.61	32.87	38.24	59.51	80.91

Table 44: Position of the supposedly best alternative in the ranking induced by the different rules (reservation levels equal to $V^I(x_{86})$ and $V^C(x_{95})$ for Itex and Cypress, respectively).

Sum

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	6.0472	37.31	54.04	62.37	68.53	72.85	84.40	92.92
ROC ΔROC	7.9753	27.93	43.04	52.48	59.31	64.77	78.52	89.00
ROC ROC	13.5755	17.99	27.16	34.33	39.79	44.18	60.78	77.16
ROC Linear	13.0248	17.70	26.73	34.37	40.52	45.63	62.26	79.40

Product

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	4.8391	40.33	55.59	64.08	69.52	74.34	87.83	96.13
ROC ΔROC	6.6139	29.03	43.65	53.06	60.43	65.89	81.23	92.47
ROC ROC	10.7798	18.39	27.72	34.99	40.61	45.23	64.44	83.68
ROC Linear	10.7942	18.17	27.40	35.34	41.53	47.34	65.77	84.42

PoP

	Average	% 1	% ≤ 2	% ≤ 3	% ≤ 4	% ≤ 5	% ≤ 10	% ≤ 20
ROC TRUE	6.1101	27.36	42.11	52.01	60.14	66.17	83.43	94.56
ROC ΔROC	7.9611	21.54	33.81	42.86	50.36	56.60	75.83	90.61
ROC ROC	11.3189	13.70	23.35	30.72	36.62	41.78	61.08	82.70
ROC Linear	11.3112	14.54	24.06	31.90	37.96	43.71	63.75	83.34

Table 45: Position of the best alternative according to the different rules in the supposedly true ranking (reservation levels equal to $V^I(x_{86})$ and $V^C(x_{95})$ for Itex and Cypress, respectively).